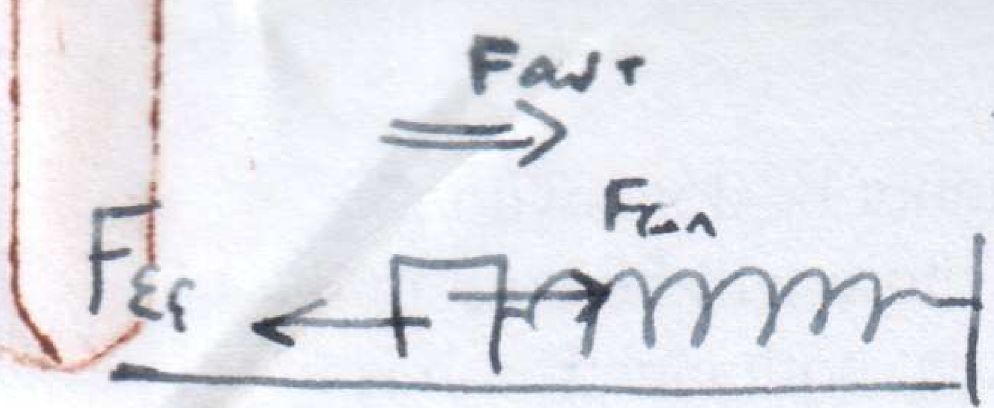
ΕΞΑΝΑΓΚΑΣΜΕΝΗ ΤΑΛΑΝΤΩΣΗ



$F_{ελ} = -kx$

$F_{αντ} = -bυ$

$\boxed{F_{εξ} = F_0 e^{j\omega t}}$ (αρμονική)

$$\Sigma F = ma \rightarrow -kx - bυ + F_0 e^{j\omega t} = m\frac{d^2x}{dt^2}$$

$$\Rightarrow m\frac{d^2x}{dt^2} + bυ + kx = F_0 e^{j\omega t}$$

$$\Rightarrow \boxed{\frac{d^2x}{dt^2} + \frac{b}{m}\frac{dx}{dt} + \frac{k}{m}x = \frac{F_0}{m}e^{j\omega t}} \quad (1)$$

Δοκιμάζω λύση $x = Ae^{j\omega t}$

$$\frac{dx}{dt} = \frac{d}{dt}\left(Ae^{j\omega t}\right) = A(j\omega)e^{j\omega t} = A(j\omega)e^{j\omega t} = j\omega x$$

$$\frac{d^2x}{dt^2} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d}{dt}\left(A(j\omega)e^{j\omega t}\right) = A(j\omega)(j\omega)e^{j\omega t} =$$

$$= A(j\omega)^2 e^{j\omega t} = Aj^2\omega^2 e^{j\omega t}$$

$$= \boxed{-A\omega^2 e^{j\omega t}} = \boxed{-\omega^2 x}$$

$$(1): \ -\omega^2 x + \frac{b}{m}j\omega x + \frac{k}{m}x = \frac{F_0}{m}e^{j\omega t}$$

$$\left(-\omega^2 + \frac{b}{m}j\omega + \frac{k}{m}\right)x = \frac{F_0}{m}e^{j\omega t}$$

$$\left(-\omega^2 + \frac{b}{m}j\omega + \frac{k}{m}\right)x = \frac{F_0}{mA}Ae^{j\omega t}$$

$$\left(-\omega^2 + \frac{b}{m}j\omega + \frac{k}{m}\right)x = \frac{F_0}{mA}x$$

$$\Rightarrow \frac{F_0}{mA} = -\omega^2 + \frac{b}{m}j\omega + \frac{k}{m} \Rightarrow A = \frac{F_0/m}{-\omega^2 + \frac{b}{m}j\omega + \frac{k}{m}}$$

$$\Rightarrow \boxed{A = \frac{F_0}{j\omega b + (k - m\omega^2)}} \Rightarrow A = \frac{-jF_0}{\omega b + j(\omega^2 m - k)} \Rightarrow \boxed{A = \frac{-jF_0}{\omega[b + j(m\omega - k/\omega)]}}$$

$$\Rightarrow \boxed{A = -jF_0 / \omega Z \quad , \; Z = \omega\left[b + j\left(m\omega - k/\omega\right)\right]} \quad ②$$

$$X = A \; e^{j(\omega t - \phi)}$$

$$A = \frac{-jF_0}{\omega|Z|}$$

$$e^{j(\omega t - \phi)} = \cos(\omega t - \phi) + j\sin(\omega t - \phi)$$

$$\left.\begin{array}{l} \\ \\ \end{array}\right\} \quad X = \frac{-jF_0}{\omega|Z|}\cos(\omega t - \phi) + j\frac{-jF_0}{\omega|Z|}\sin(\omega t - \phi)$$

$$\Rightarrow X = -j^2\frac{F_0}{\omega|Z|}\sin(\omega t - \phi) + \frac{-jF_0}{\omega|Z|}\cos(\omega t - \phi)$$

$$\Rightarrow X = -(-1)\frac{F_0}{\omega|Z|}\sin(\omega t - \phi) + \frac{-jF_0}{\omega|Z|}\cos(\omega t - \phi)$$

$$\Rightarrow \boxed{X = \frac{F_0}{\omega|Z|}\sin(\omega t - \phi) - j\frac{F_0}{\omega|Z|}\cos(\omega t - \phi)}$$

$$F_{e\varsigma} = F_0 \, e^{j\omega t} \Rightarrow \boxed{F_{e\varsigma} = F_0\cos\omega t + jF_0\sin\omega t}$$

$$Re(F_{e\varsigma}) = F_0\cos\omega t = F$$

$$Re(X) = \frac{F_0}{\omega|Z|}\sin(\omega t - \phi) = x$$

$$v = \frac{dx}{dt} = \frac{F_0}{\omega|Z|}\left[\sin(\omega t - \phi)\right]' =$$

$$= \frac{F_0}{\omega|Z|}\cos(\omega t - \phi)(\omega t - \phi)'$$

$$= \frac{F_0}{\cancel{\omega}|Z|}\cdot\cancel{\omega}\cos(\omega t - \phi) = \frac{F_0}{|Z|}\cos(\omega t - \phi)$$

$$\boxed{|Z| = \sqrt{(b\omega)^2 + \left(m\omega - \frac{k}{\omega}\right)^2}}$$

$$\boxed{\begin{array}{l} Z = b\omega + j\left(m\omega - \frac{k}{\omega}\right) \\ Z = |Z| e^{j\phi} \end{array}}$$

$$X = A e^{j\omega t}$$

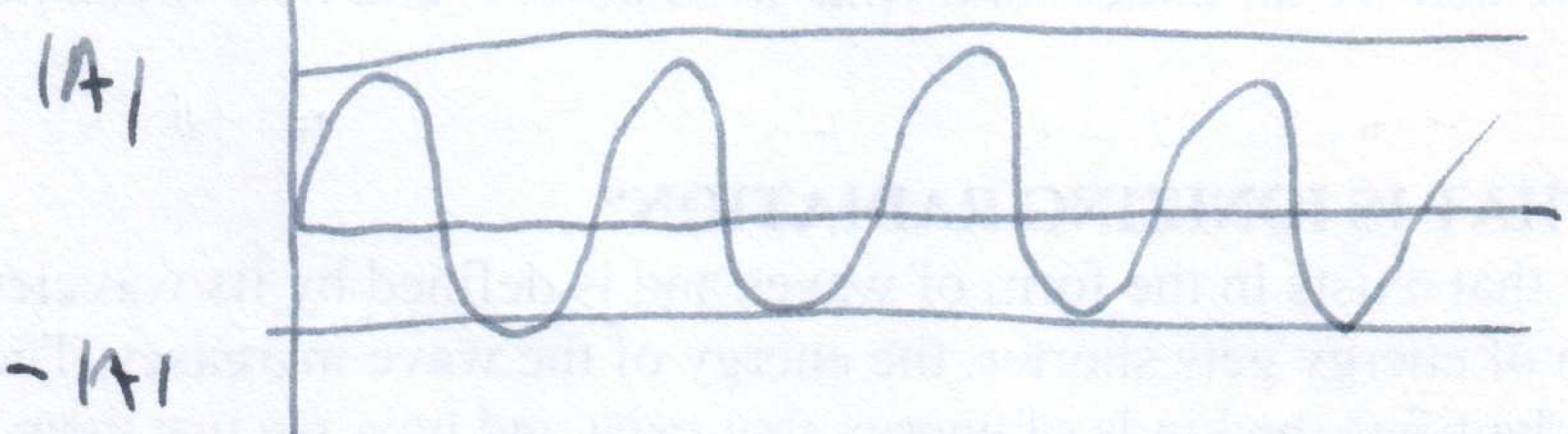
$$A = \frac{jF_0}{\omega Z}$$

$$X = \frac{-jF_0}{\omega Z} e^{j\omega t}$$

$$\xrightarrow{Z = |Z| e^{j\phi}} X = \frac{-jF_0}{\omega |Z| e^{j\phi}} e^{j\omega t}$$

$$\Rightarrow \boxed{X = \frac{-jF_0}{\omega |Z|} e^{j(\omega t - \phi)}}$$

$$X = |A| e^{j(\omega t - \phi)}$$



Το σώμα εκτελεί ΑΑΤ με πλάτος $|A| = \frac{jF_0}{\omega |Z|}$

$$\alpha = \frac{dv}{dt} = \frac{F_0}{|Z|}\Big(\cos(\omega t - \phi)\Big)'$$

$$= \frac{F_0}{|Z|}\Big[-\sin(\omega t-\phi)\Big](\omega t-\phi)'$$

$$= -\frac{F_0\,\omega}{|Z|}\sin(\omega t - \phi)$$

$$\boxed{\begin{aligned}
f &= Re(F_{cl}) = F_0\cos\omega t \\
x &= Re(x) = \frac{F_0}{\omega |Z|}\sin(\omega t - \phi) \quad (\phi \text{ πίσω ως το } F) \\
v &= \frac{dx}{dt} = \frac{F_0}{|Z|}\cos(\omega t - \phi) \\
&= \frac{F_0}{|Z|}\sin\left(\omega t - \phi + \frac{\pi}{2}\right) \quad \left(\frac{\pi}{2} \text{ μπροστά ως το } x\right) \\
\alpha &= \frac{dv}{dt} = \frac{\omega F_0}{|Z|}\big(-\sin(\omega t - \phi)\big) \\
&= \frac{\omega F_0}{|Z|}\sin(\omega t - \phi + \pi) \quad (\pi \text{ μπροστά ως το } x) \\
|Z| &= \sqrt{(b)^2 + \left(m\omega - \frac{k}{\omega}\right)^2}
\end{aligned}}$$

$$|A| = \frac{F_0}{\omega |Z|}$$

Αλ. λύση

$$\boxed{A = \frac{F_0}{\sqrt{b^2\omega^2 + (m\omega^2 - k)^2}}}$$

$$v_{max} = \omega |A| = \frac{F_0}{|Z|}$$

$$\boxed{A = \frac{F_0}{\omega\sqrt{b^2 + \left(m\omega - \frac{k}{\omega}\right)^2}}}$$

$$|Z| = \sqrt{(b)^2 + \left(m\omega - \frac{k}{\omega}\right)^2}$$

W_0: συχνότητα συντονισμού

$$Z(W_0) = b$$

$$m W_0 = \frac{k}{W_0} \Rightarrow m W_0 - \frac{k}{W_0} = 0$$

$$|Z| = \sqrt{b^2 + \left(m W_0 - \frac{k}{W_0}\right)^2} = \sqrt{b^2 + 0} = \sqrt{b^2} = b$$

$$m W_0 = \frac{k}{W_0} \Rightarrow m W_0^2 = k \Rightarrow W_0^2 = \frac{k}{m} \Rightarrow \boxed{W_0 = \sqrt{\frac{k}{m}}}$$

W_0 είναι η συχνότητα της επιβαλλόμενης ταλάντωσης (ιδιοσυχνότητα)

$$|Z| = min \Rightarrow |A| = max \Rightarrow \frac{d|A|}{dw} = 0$$

(I)

$$|A| = \frac{F_0}{\omega |Z|} \qquad U_{max} = \omega |A| \quad \Rightarrow \quad U_{max} = \omega \frac{F_0}{|Z|} \quad \Rightarrow \quad \boxed{U_{max} = \frac{F_0}{|Z|}}$$

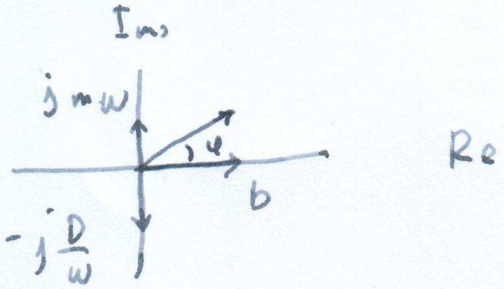
$$V = \dot{x} \qquad x = \frac{-jF_0}{\omega Z} e^{j(\omega t - \varphi)} \quad \Rightarrow \quad V = j\cancel{\omega} \frac{-jF_0}{\cancel{\omega} Z} e^{j(\omega t - \varphi)}$$

$$\Rightarrow \quad V = \frac{-j^2 F_0}{Z} e^{j(\omega t - \varphi)}$$

$$\Rightarrow \quad \boxed{V = \frac{F_0}{Z} e^{j(\omega t - \varphi)}}$$

$$\boxed{Z = b + j\left(m\omega - \frac{D}{\omega}\right)} \qquad \boxed{\tan\varphi = \frac{m\omega - \frac{D}{\omega}}{b}}$$

$$\boxed{|Z| = \sqrt{b^2 + \left(m\omega - \frac{D}{\omega}\right)^2}}$$



$$f = Re(F_{f\varphi}) = F_0 \cos\omega t \qquad 0$$

$$x = Re(x) = \frac{F_0}{\omega |Z|} \sin(\omega t - \varphi) \qquad -\varphi - \pi/2 \qquad [-jF_0]$$

$$\upsilon = Re(v) = \frac{F_0}{|Z|} \cos(\omega t - \varphi) \qquad -\varphi$$

$$\alpha = \frac{\omega F_0}{|Z|}\left[-\sin(\omega t - \varphi)\right] = \frac{j^2 \omega F_0}{|Z|} \sin(\omega t - \varphi) \qquad -\varphi + \pi/2 \qquad [j^2]$$

ΔΙΑΦΟΡΑ ΦΑΣΗΣ U και F

$$\boxed{\tan\varphi = \frac{m\omega - \frac{D}{\omega}}{b}}$$

$$\boxed{\begin{gathered} U = \frac{F_0}{|Z|}\cos(\omega t - \varphi) \\ F = F_0 \cos\omega t \end{gathered}}$$

$$\boxed{Z = b + j\left(m\omega - \frac{D}{\omega}\right)}$$

▷ ΥΨΗΛΕΣ ΣΥΧΝΟΤΗΤΕΣ

- $m\omega > \frac{D}{\omega} \Rightarrow \tan\varphi > 0 \Rightarrow \varphi > 0 \Rightarrow$ Η ΤΑΧΥΤΗΤΑ ΥΣΤΕΡΕΙ της F

  - $\omega \to \infty \Rightarrow \tan\varphi \to \infty \Rightarrow \varphi \to \pi/2 \Rightarrow$

    $\Rightarrow$ Η ΤΑΧΥΤΗΤΑ ΥΣΤΕΡΕΙ ΤΗΣ F ΚΑΤΑ $\pi/2$

- $m\omega > \frac{D}{\omega} \Rightarrow$ ο ορος $m\omega$ κυριαρχει στην αποκριση

  $\Rightarrow$ Η ΕΜΠΕΔΗΣΗ ΕΧΕΙ ΑΔΡΑΝΕΙΑΚΗ ΣΥΜΠΕΡΙΦΟΡΑ

▷ ΧΑΜΗΛΕΣ ΣΥΧΝΟΤΗΤΕΣ

- $m\omega < \frac{D}{\omega} \Rightarrow \tan\varphi < 0 \Rightarrow \varphi < 0 \Rightarrow$ Η ΤΑΧΥΤΗΤΑ ΠΡΟΗΓΕΙΤΑΙ της F

  - $\omega \to 0 \Rightarrow \frac{D}{\omega} \to \infty \Rightarrow \tan\varphi \to -\infty \Rightarrow \varphi \to -\frac{\pi}{2}$

    $\Rightarrow$ Η ΤΑΧΥΤΗΤΑ ΠΡΟΗΓΕΙΤΑΙ ΤΗΣ F ΚΑΤΑ $\pi/2$

- $m\omega < \frac{D}{\omega} \Rightarrow$ ο όρος $\frac{D}{\omega}$ κυριαρχει στην αποκριση

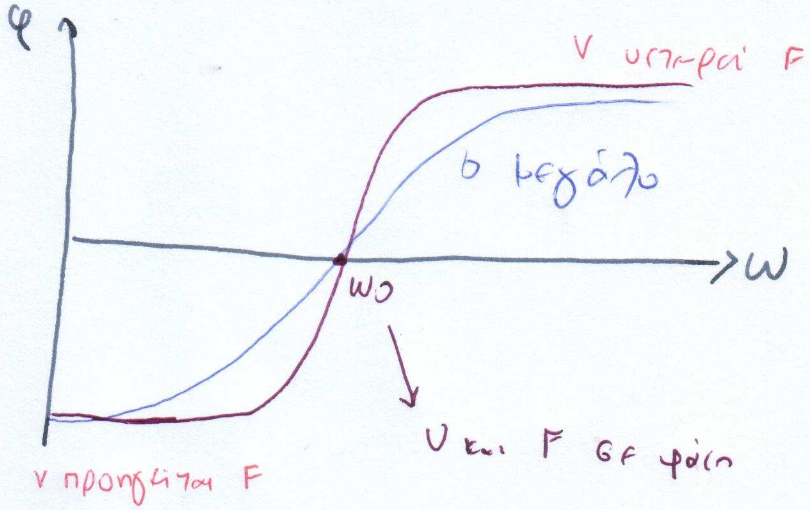
  $\Rightarrow$ Η ΕΜΠΕΔΗΣΗ ΕΧΕΙ ΕΛΑΣΤΙΚΗ ΣΥΜΠΕΡΙΦΟΡΑ

▷ ΣΥΝΤΟΝΙΣΜΟΣ

- $m\omega_0 = \frac{D}{\omega_0} \Rightarrow D = m\omega_0^2 \Rightarrow \omega_0 = \sqrt{\frac{D}{m}}$

  $\Rightarrow \varphi = 0 \Rightarrow$ Η ΤΑΧΥΤΗΤΑ ΕΙΝΑΙ ΣΕ ΦΑΣΗ ΜΕ ΤΗΝ F

φασική γωνία
φ
μετάξυ υ και F



ΔΙΑΦΟΡΑ ΦΑΣΗΣ X ΚΑΙ F

$$F = F_0 \cos\omega t$$

$$X = \frac{F_0}{\omega|Z|}\sin(\omega t - \varphi)$$

$$X = \frac{-jF_0}{\omega Z} e^{j(\omega t - \varphi)}$$ ⇒ $-j$ ⇒ $-\pi/2$ πίσω από F

ΟΛΙΚΗ ΓΩΝΙΑ ΦΑΣΗΣ ΘΕΣΗΣ ΔΥΝΑΜΗΣ

$$\text{ΟΓΦ} = -\varphi - \frac{\pi}{2}$$

▷ ΥΨΗΛΕΣ ΣΥΧΝΟΤΗΤΕΣ

$\tan\varphi \to 0 \Rightarrow \varphi > 0 \Rightarrow \varphi + \pi/2 > \pi/2$

⇒ Η ΜΕΤΑΤΟΠΙΣΗ ΥΣΤΕΡΑ ΤΗΣ F ΑΦΟΥ
ΚΑΙ Η V ΥΣΤΕΡΑ ΤΗΣ F

$\omega \to \infty \Rightarrow \tan\varphi \to \infty \Rightarrow \varphi \to \pi/2 \Rightarrow -\varphi - \frac{\pi}{2} = -\frac{\pi}{2} - \frac{\pi}{2} = -\pi$

$\Rightarrow \text{ΟΓΦ} = -\pi$ ⇒ Η ΜΕΤΑΤΟΠΙΣΗ ΥΣΤΕΡΑ ΤΗΣ F
ΚΑΤΑ Π ΑΝΤΙΚΡΙ

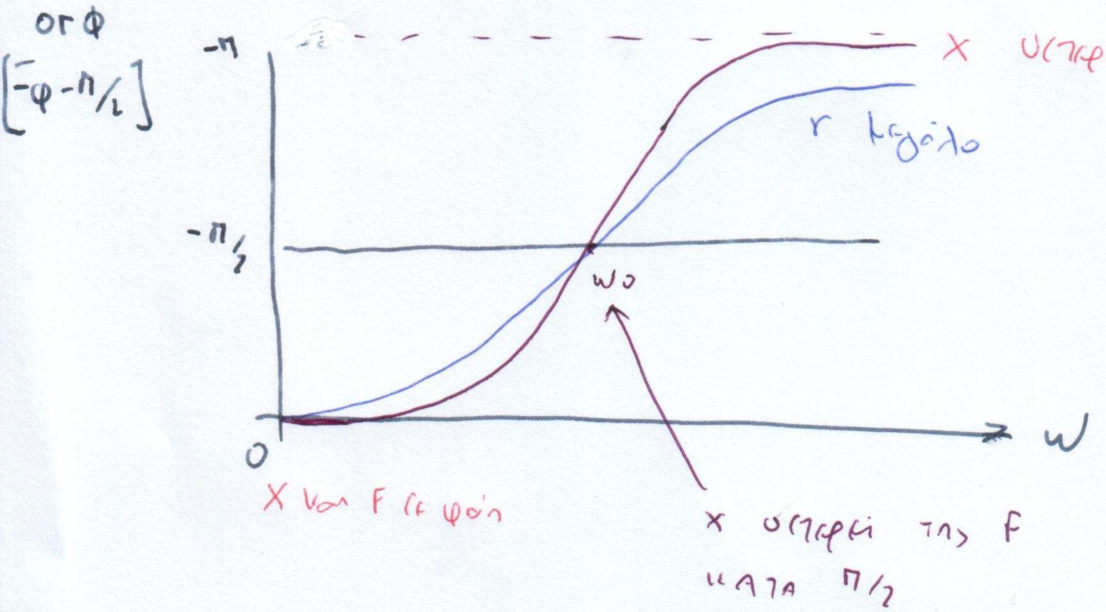
▷ ΧΑΜΗΛΕΣ ΣΥΧΝΟΤΗΤΕΣ

$$\tan\varphi < 0 \Rightarrow \varphi < 0 \Rightarrow -\varphi > 0 \Rightarrow -\varphi - \frac{\pi}{2} > -\frac{\pi}{2} \Rightarrow \boxed{\text{ΟΓΦ} > -\frac{\pi}{2}}$$

$$\omega \to 0 \Rightarrow \tan\varphi \to -\infty \Rightarrow \varphi \to -\frac{\pi}{2} \Rightarrow -\varphi - \frac{\pi}{2} \Rightarrow \frac{\pi}{2} - \frac{\pi}{2}$$

$$\Rightarrow -\varphi - \frac{\pi}{2} \to 0 \Rightarrow \text{ΟΓΦ} \to 0$$

⇒ Η ΜΕΤΑΤΟΠΙΣΗ ΚΑΙ Η ΔΥΝΑΜΗ ΕΙΝΑΙ ΣΕ ΦΑΣΗ



ΕΞΑΡΤΗΣΗ ⟨ΜΕΤΡΟΥ ΤΑΧΥΤΗΤΑΣ / ΘΕΣΗΣ⟩ ΚΑΙ ΣΥΧΝΟΤΗΤΑΣ

▷ ΤΑΧΥΤΗΤΑ

$$V = \frac{F_0}{Z} e^{j(\omega t - \varphi)} \Rightarrow |V| = \frac{F_0}{|Z|}$$

$$\Rightarrow \boxed{|V| = \frac{F_0}{\sqrt{b^2 + \left(m\omega - \frac{D}{\omega}\right)^2}}}$$

ΣΥΝΤΟΝΙΣΜΟΣ

Όταν $\boxed{\omega = \omega_0} \Rightarrow m\omega_0 = \frac{D}{\omega_0} \Rightarrow$ ο όρος $\left(m\omega_0 - \frac{D}{\omega_0}\right) = 0$

$$\Rightarrow |V| = \frac{F_0}{\sqrt{b^2}} \Rightarrow \boxed{|V| = \frac{F_0}{b}}$$

ΥΨΗΛΕΣ ΣΥΧΝΟΤΗΤΕΣ

Η ΕΜΠΕΔΗΣΗ ΕΧΕΙ ΑΔΡΑΝΕΙΑΚΗ ΣΥΜΠΕΡΙΦΟΡΑ

$$\Leftrightarrow m\omega > \frac{D}{\omega} \Rightarrow |V| \sim \frac{F_0}{\sqrt{b^2 + (m\omega)^2}} \sim \frac{F_0/b}{m\omega}$$

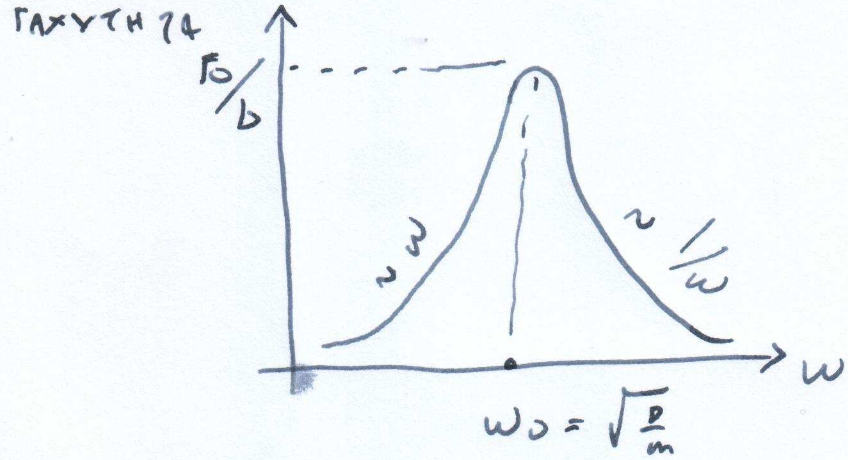
$$\Rightarrow \boxed{|V| \sim \frac{1}{\omega}} \Rightarrow |V| \text{ μειώνεται με το } \omega$$

ΧΑΜΗΛΕΣ ΣΥΧΝΟΤΗΤΕΣ

Η ΕΜΠΕΔΗΣΗ ΕΧΕΙ ΕΛΑΣΤΙΚΗ ΣΥΜΠΕΡΙΦΟΡΑ

$$\Leftrightarrow m\omega < \frac{D}{\omega} \Rightarrow |V| \sim \frac{F_0}{\sqrt{b^2 + \left(\frac{D}{\omega}\right)^2}} \sim \frac{F_0/b}{D} \cdot \omega$$

$$\Rightarrow \boxed{|V| \sim \omega} \Rightarrow |V| \text{ αυξάνεται με το } \omega$$



ΘΕΣΗ

$$x = \frac{-j F_0}{\omega Z} e^{j(\omega t - \varphi)} \Rightarrow |x| = \frac{F_0}{\omega |Z|}$$

$$\Rightarrow \boxed{|x| = \frac{F_0}{\omega \sqrt{b^2 + \left(m\omega - \frac{D}{\omega}\right)^2}} = A}$$

$$\Rightarrow A = \frac{F_0}{\sqrt{\omega^2 b^2 + \omega^2 \left(m\omega - \frac{D}{\omega}\right)^2}} = \frac{F_0}{\sqrt{\omega^2 b^2 + \left(\omega m \omega - \cancel{\omega}\frac{D}{\cancel{\omega}}\right)^2}}$$

$$\Rightarrow \boxed{\ = \frac{F_0}{\sqrt{(\omega b)^2 + (m\omega^2 - D)^2}}}$$

$$\frac{dA}{dw} = F_0 \frac{d}{dw}\left[(bw)^2 + (mw^2-k)^2\right]^{-3/2} \quad , \quad u = (bw)^2 + (mw^2-k)^2$$

$$\Rightarrow \frac{du^{-3/2}}{du}\frac{du}{dw} = 0 \Rightarrow -\frac{1}{2}u^{-3/2}\frac{du}{dw} = 0 \xRightarrow{u \neq 0} \frac{du}{dw} = 0$$

$$\Rightarrow \frac{d}{dw}\left[(bw)^2 + (mw^2-k)^2\right] = 0 \Rightarrow \frac{d}{dw}(bw)^2 + \frac{d}{dw}(mw^2-k)^2 = 0$$

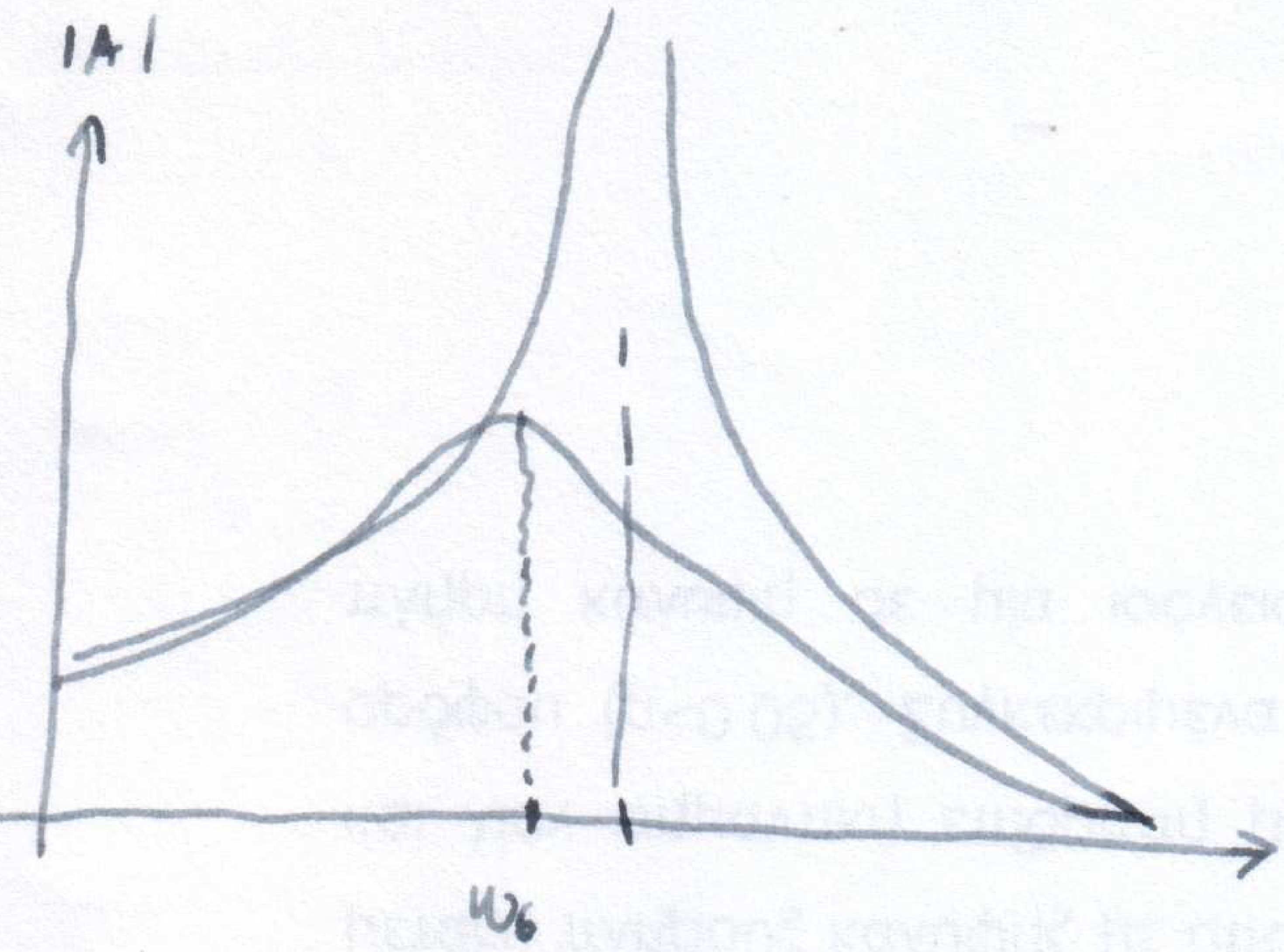
$$\Rightarrow 2bw(bw)' + 2(mw^2-k)(mw^2-k)' = 0$$

$$\Rightarrow 2bw\,b + 2(mw^2-k)\,2mw = 0 \Rightarrow$$

$$\Rightarrow 2w\left(b^2 + 2(mw^2-k)\,m\right) = 0 \Rightarrow b^2 + 2m(mw^2-k) = 0$$

$$\Rightarrow b^2 + 2m^2w^2 - 2mk = 0 \Rightarrow 2m^2w^2 = 2mk - b^2 \Rightarrow$$

$$\Rightarrow w^2 = \frac{\cancel{2}mk}{\cancel{2}m^2} - \frac{b^2}{2m^2} \Rightarrow \boxed{w^2 = \frac{k}{m} - \frac{b^2}{2m^2}}$$



$\boxed{b=0}$

$$\omega_6^2 = \frac{k}{m} - \frac{b^2}{2m^2} \Rightarrow \omega_6^2 = \frac{k}{m}$$

$$\frac{k}{m} = \omega_0^2$$

$$\omega_6^2 = \omega_0^2 \Rightarrow \boxed{\omega_6 = \omega_0}$$

$\boxed{b \neq 0}$

$$\omega_6 = \frac{k}{m} - \frac{b^2}{2m^2} \Rightarrow \boxed{\omega_6 = \omega_0 - \frac{b^2}{2m^2}}$$