②

RLC :

$b \to R$

$m = L$

$D = \frac{1}{C}$

$F_0 \Rightarrow V_0$

$A \to Q$

$$\boxed{Q = \frac{-j V_0}{\omega \left[R + j\left(L\omega - \frac{1}{\omega C} \right) \right]}}$$

$$\boxed{Z = R + j\left(L\omega - \frac{1}{\omega C} \right)}$$

$$|Z| = \sqrt{R^2 + \left(L\omega - \frac{1}{\omega C} \right)^2}$$

Εν τέλει

$x = A e^{j\omega t}$ ή $x = A e^{j(\omega t - \varphi)}$

$$A = \frac{-j F_0}{\omega Z}$$

$$x = \frac{-j F_0}{\omega Z} \left[\cos(\omega t - \varphi) + j \sin(\omega t - \varphi) \right]$$

$$e^{j(\omega t - \varphi)} = j \sin(\omega t + \varphi) + \cos(\omega t - \varphi)$$

$$\rightarrow \boxed{x = \frac{-j F_0}{Z} \cos(\omega t - \varphi) + \frac{F_0}{Z} \sin(\omega t - \varphi)}$$

$$Re\{x\} = \frac{F_0}{|Z|} \sin(\omega t - \varphi) \qquad Re\{F_{ext}\} = F_0 \cos\omega t$$

$$Im\{x\} = \frac{F_0}{|Z|} \cos(\omega t - \varphi) \qquad Im\{F_{ext}\} = F_0 \sin\omega t$$

$$U = \frac{dx}{dt} = \dot{x} \Rightarrow U = \frac{d}{dt}\left(-j\frac{F_0}{\omega|Z|}e^{j(\omega t-\varphi)}\right) = -j\frac{F_0}{\omega|Z|}\, j\omega\, e^{j(\omega t-\varphi)}$$

$$= \frac{F_0}{\omega|Z|}\,\omega\, e^{j(\omega t-\varphi)} = \frac{F_0}{|Z|}\, e^{j(\omega t-\varphi)}$$

$$\Rightarrow \boxed{\dot{x} = \frac{F_0}{|Z|}e^{j(\omega t-\varphi)}} \qquad \boxed{\dot{x} = \frac{F_0}{|Z|}\cos(\omega t-\varphi) + j\frac{F_0}{|Z|}\sin(\omega t-\varphi)}$$

$$e^{j\omega t} = \cos\omega t + j\sin\omega t$$

$$x = -\frac{jF_0}{\omega|Z|}\cos\omega t + \frac{F_0}{|Z|}\sin\omega t$$

$$\Rightarrow \boxed{x = \frac{F_0}{\omega|Z|}\sin(\omega t-\varphi) - j\frac{F_0}{\omega|Z|}\cos(\omega t-\varphi)}$$

$$Re\{\dot{x}\} = \frac{F_0}{|Z|}\cos(\omega t-\varphi) = \frac{F_0}{|Z|}\sin\left(\omega t+\frac{\pi}{2}-\varphi\right)$$

$$Re\{x\} = \frac{F_0}{\omega|Z|}\sin(\omega t-\varphi)$$

Άρα η ταχύτητα U είναι $\frac{\pi}{2}$ μπροστά από τη δύναμη F
και το x

Άρα το $i = \frac{dQ}{dt} = \dot{Q}$ είναι $\frac{\pi}{2}$ μπροστά από την τάση του πυκνωτή δηλαδή το i είναι $\frac{\pi}{2}$ μπροστά από την τάση του πυκνωτή

I (R)

v_c (Z_c)

ΕΠΙΤΑΧΥΝΣΗ

$$\left.\begin{aligned} \alpha &= \frac{dv}{dt} \\ v = \dot{x} &= \frac{F_0}{|Z|} e^{j(\omega t-\varphi)} \end{aligned}\right\} \quad \alpha = \frac{d}{dt}\left(\frac{F_0}{|Z|} e^{j(\omega t+\varphi)}\right) \Rightarrow \boxed{\alpha = \frac{j\omega F_0}{|Z|} e^{j(\omega t+\varphi)}}$$

$$\boxed{x = \frac{-j\,F_0}{|Z|} e^{j(\omega t-\varphi)}}$$

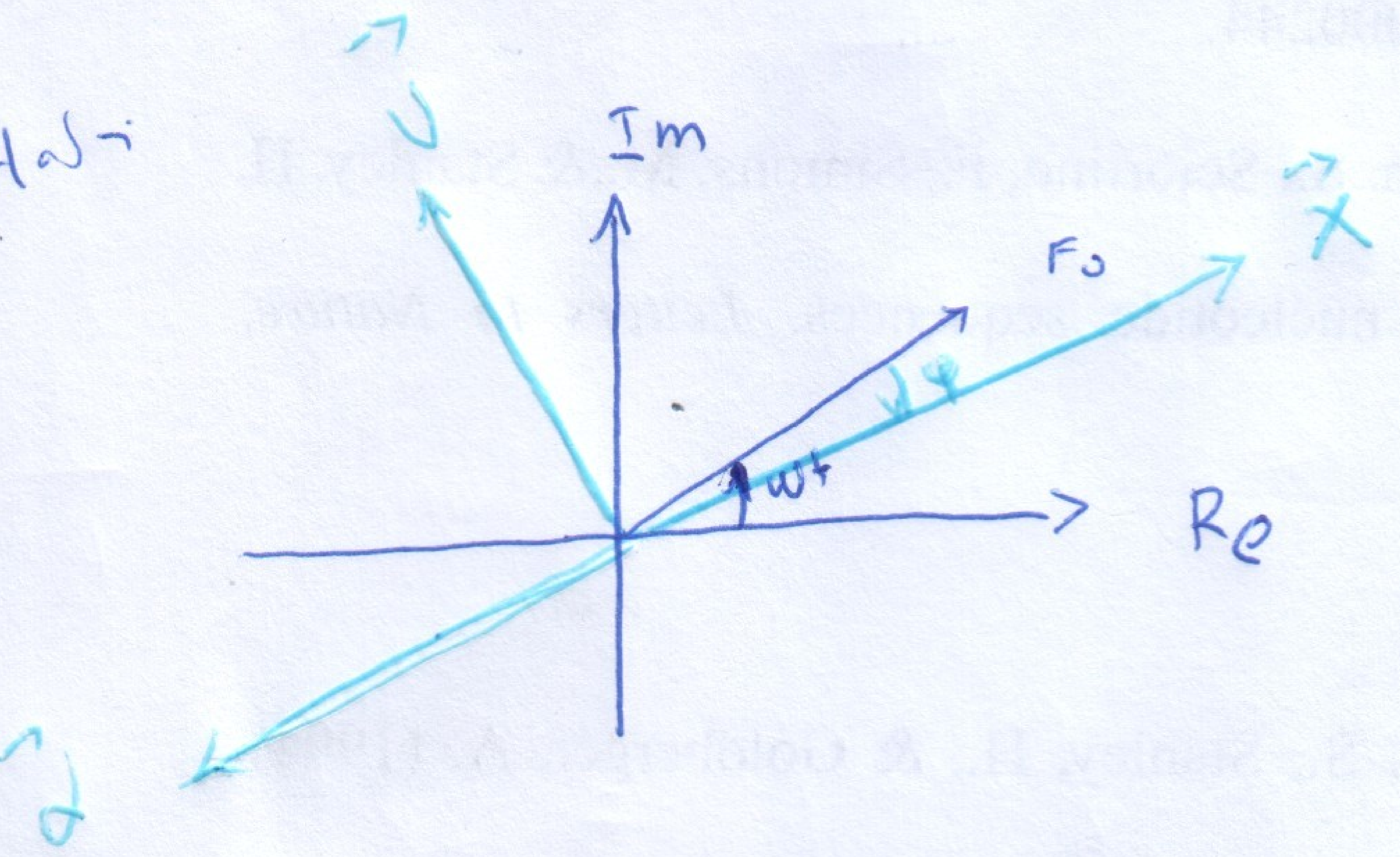
$$\boxed{\dot{x} = \frac{F_0}{|Z|} e^{j(\omega t-\varphi)}}$$

Αλλα ισχυει κινησης ο

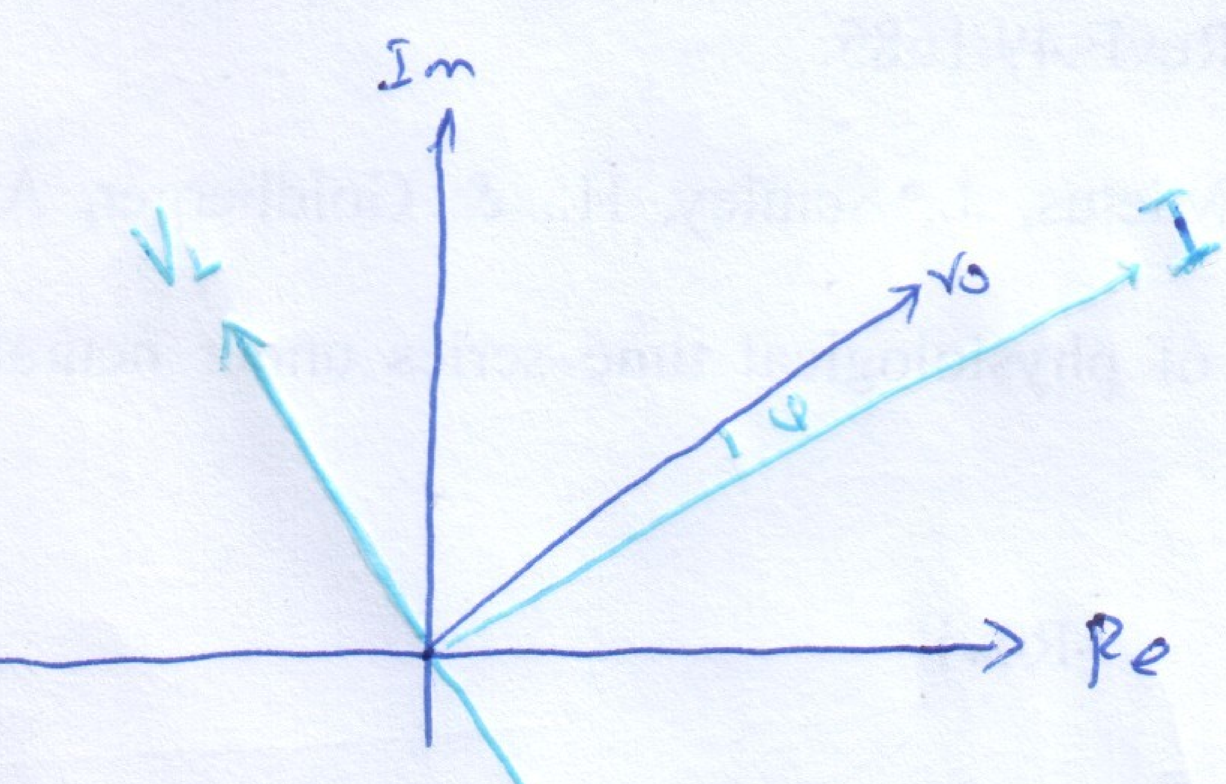
Επομένως

> Το x είναι $-j\left(-\frac{\pi}{2}\right)$ πριν από το $\dot{x}$ και
>
> Το $\ddot{x}$ είναι $+j\left(\frac{\pi}{2}\right)$ σε σχέση με το $\dot{x}$

Διαγρ.



RLC



ΜΗΧΑΝΙΚΕΣ - RLC

$$|A| = \frac{F_0}{\omega|Z|} \qquad I = \frac{V_0}{|Z|}$$

$$|Z| = \sqrt{b^2 + \left(m\omega - \frac{D}{\omega}\right)^2} \qquad |Z| = \sqrt{R^2 + \left(L\omega - \frac{1}{\omega C}\right)} = \sqrt{R^2 - (Z_L - Z_C)^2}$$

$$x = A e^{j(\omega t - \varphi)} = \frac{-jF_0}{|Z|} e^{j(\omega t - \varphi)} \qquad V_C = \frac{Q}{C} = \frac{-jV_0}{C|Z|} e^{j(\omega t - \varphi)}$$

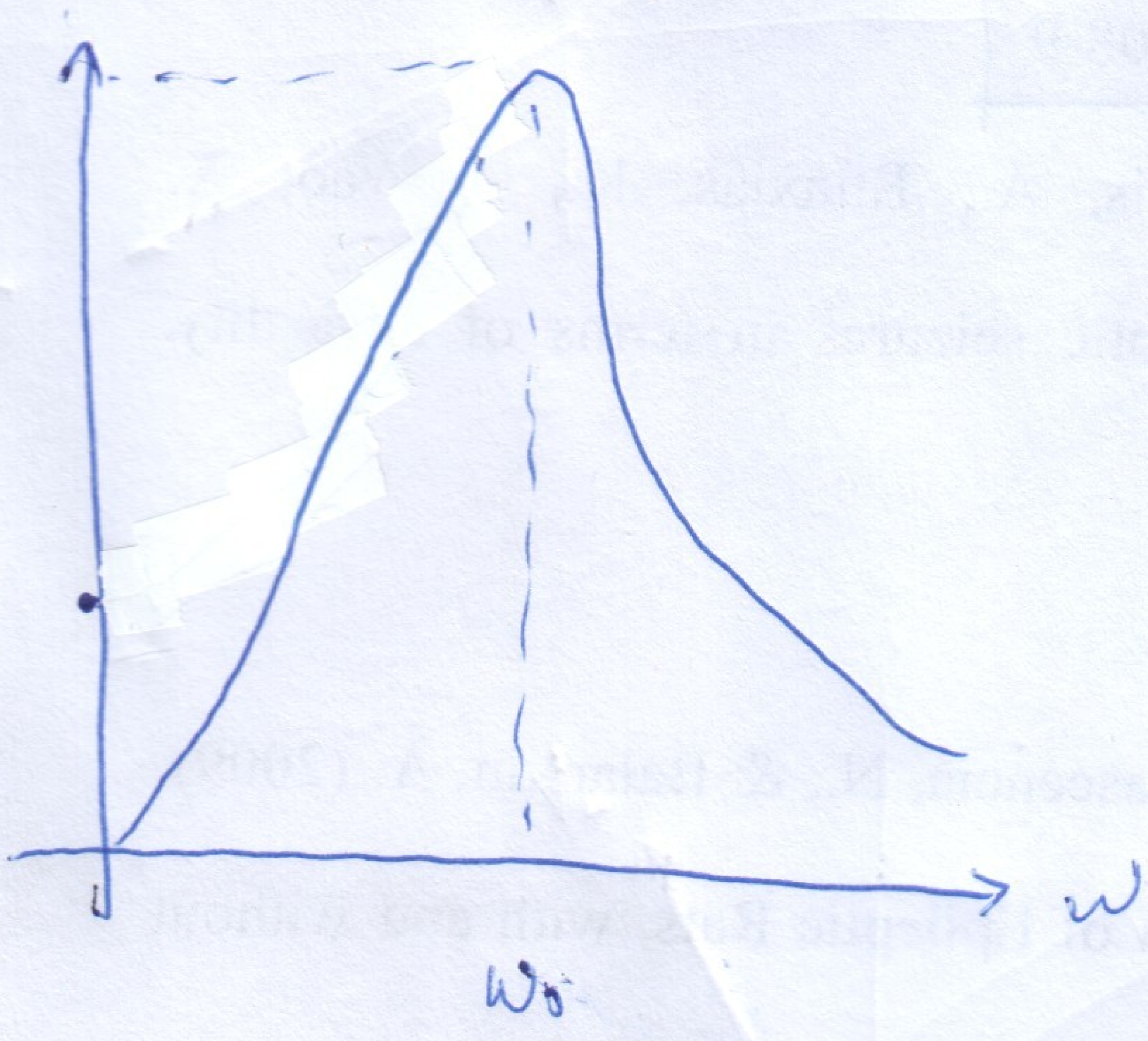
$$\dot{x} = \frac{F_0}{|Z|} e^{j(\omega t - \varphi)} \qquad i = \frac{V_0}{|Z|} e^{j(\omega t - \varphi)}$$

$$\ddot{x} = \frac{j\omega F_0}{|Z|} e^{j(\omega t - \varphi)} \qquad V_L = -L\frac{di}{dt} = -\frac{j\omega V_0}{L|Z|} e^{-(j\omega t - \varphi)}$$

ΣΥΝΤΟΝΙΣΜΟΣ

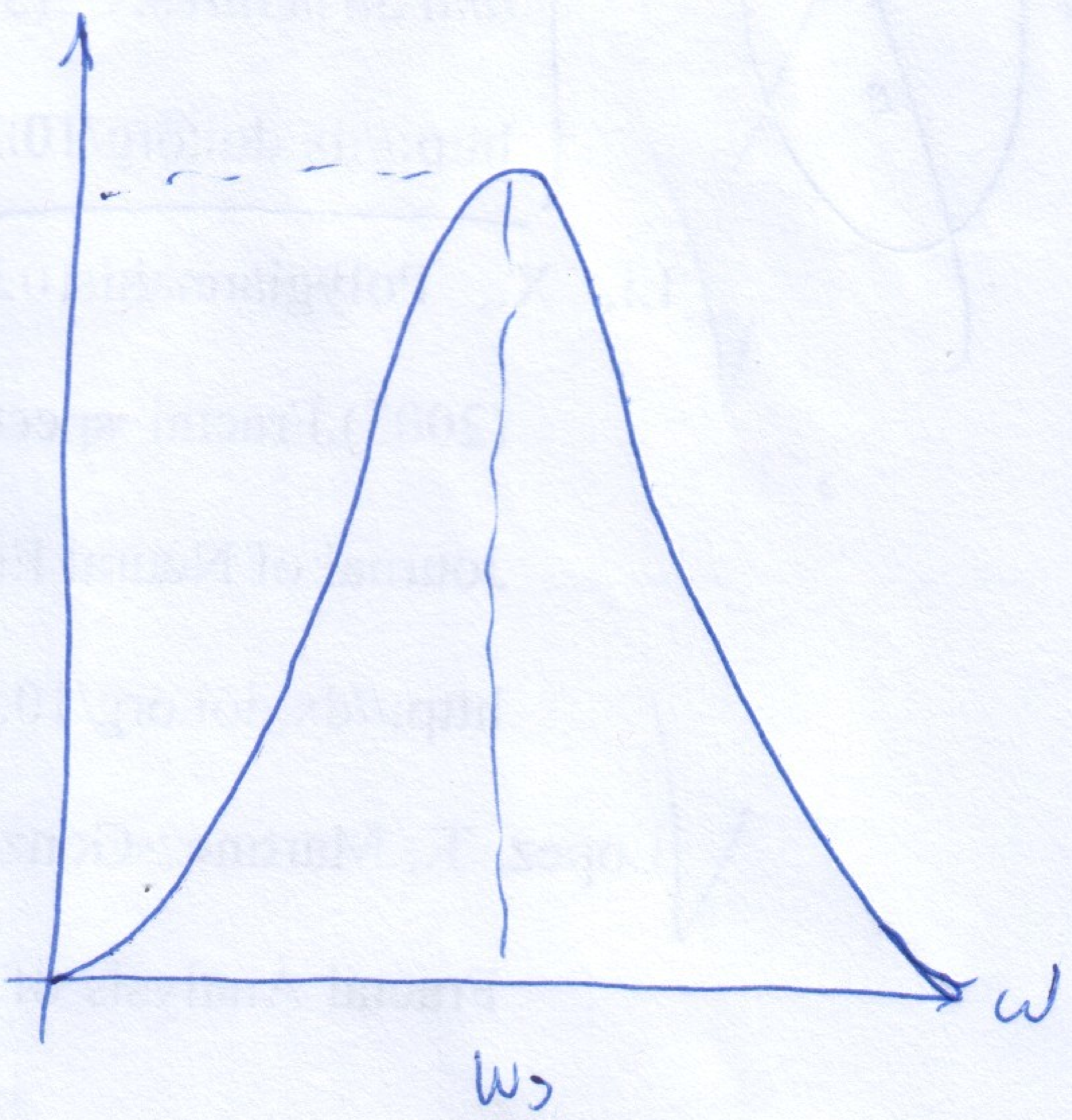
ΤΑΧΥΤΗΤΑ

$$\omega|A| = \frac{F_0}{b}$$

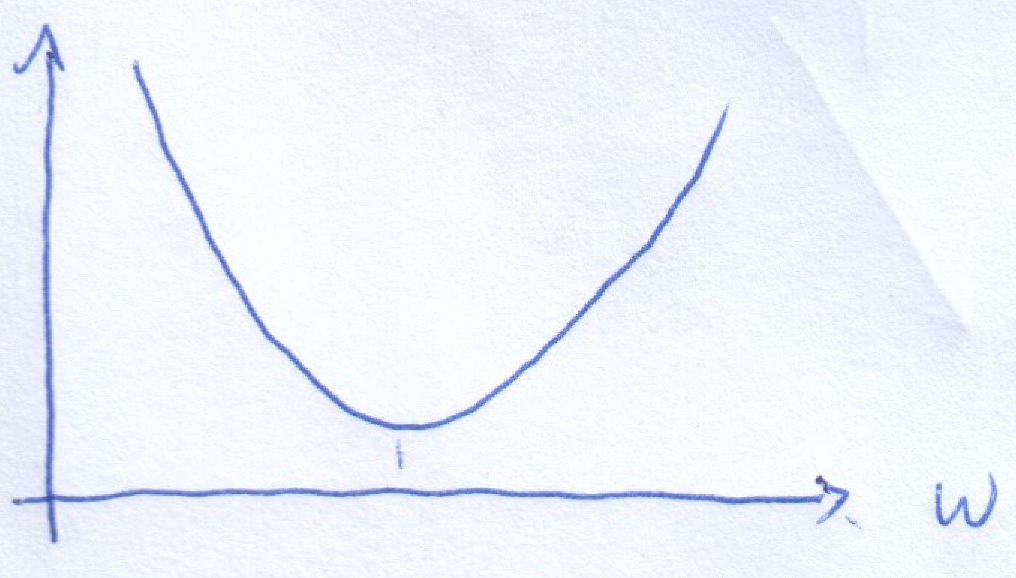


ΡΕΥΜΑ

$$I_0 = \frac{V_0}{R}$$

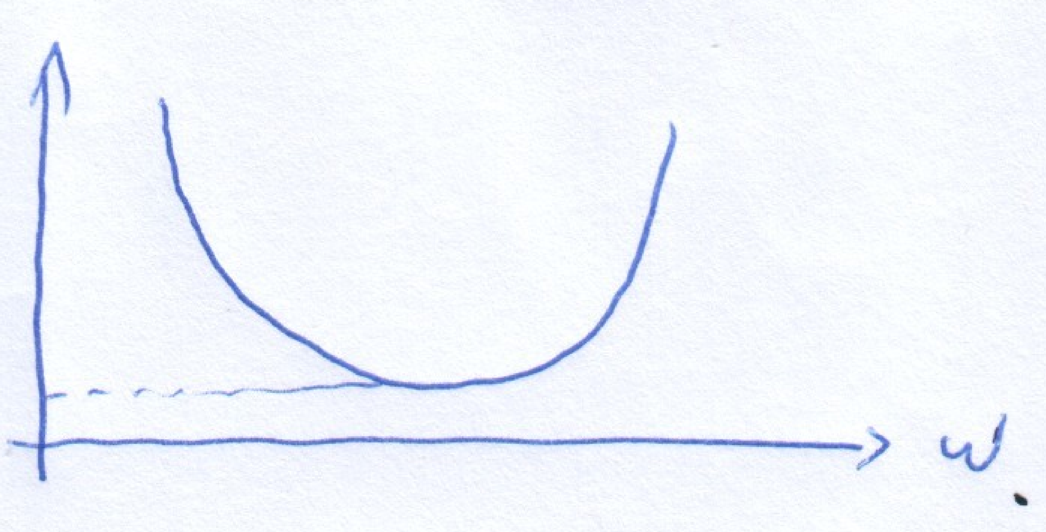


|Z|

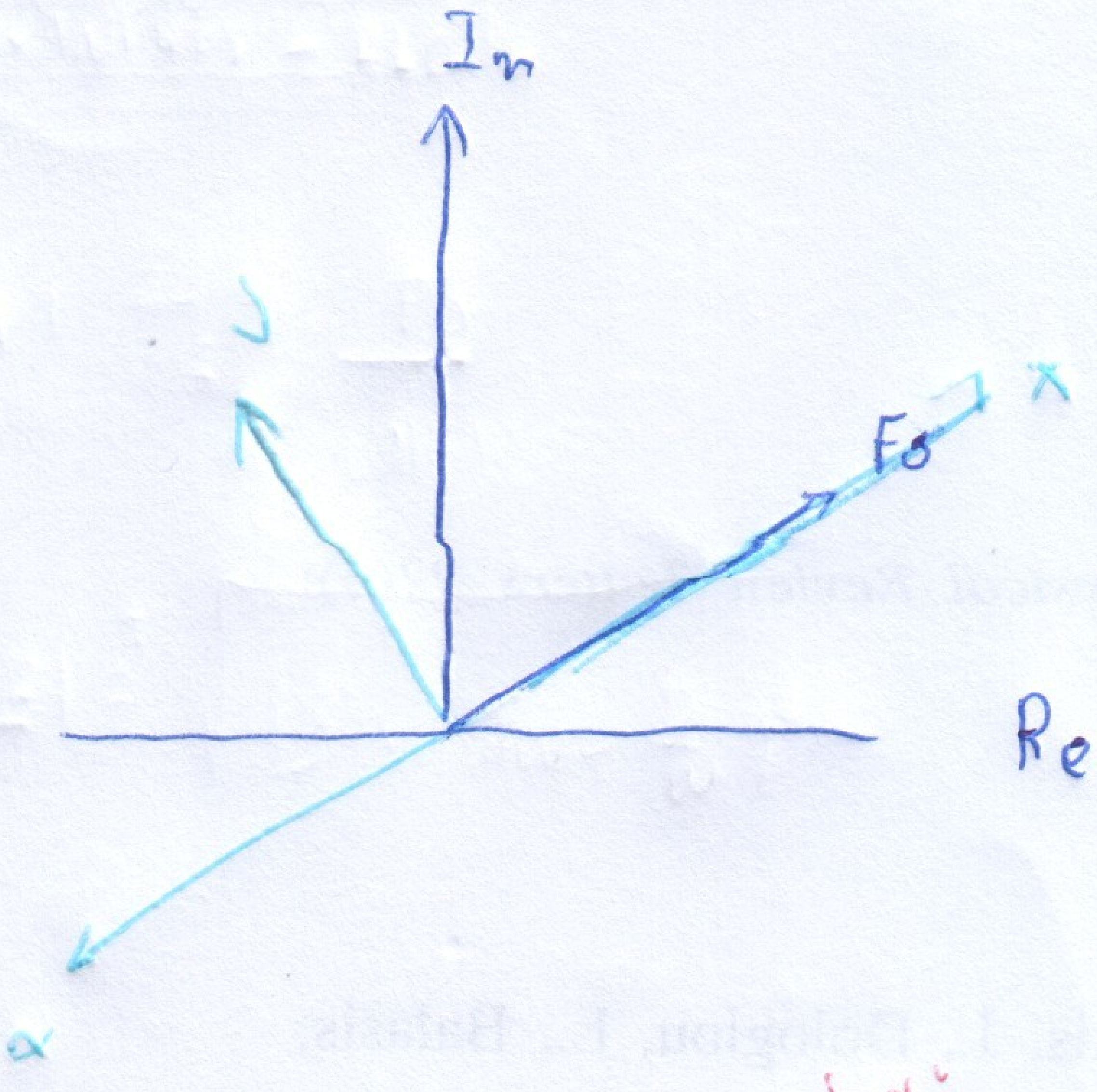


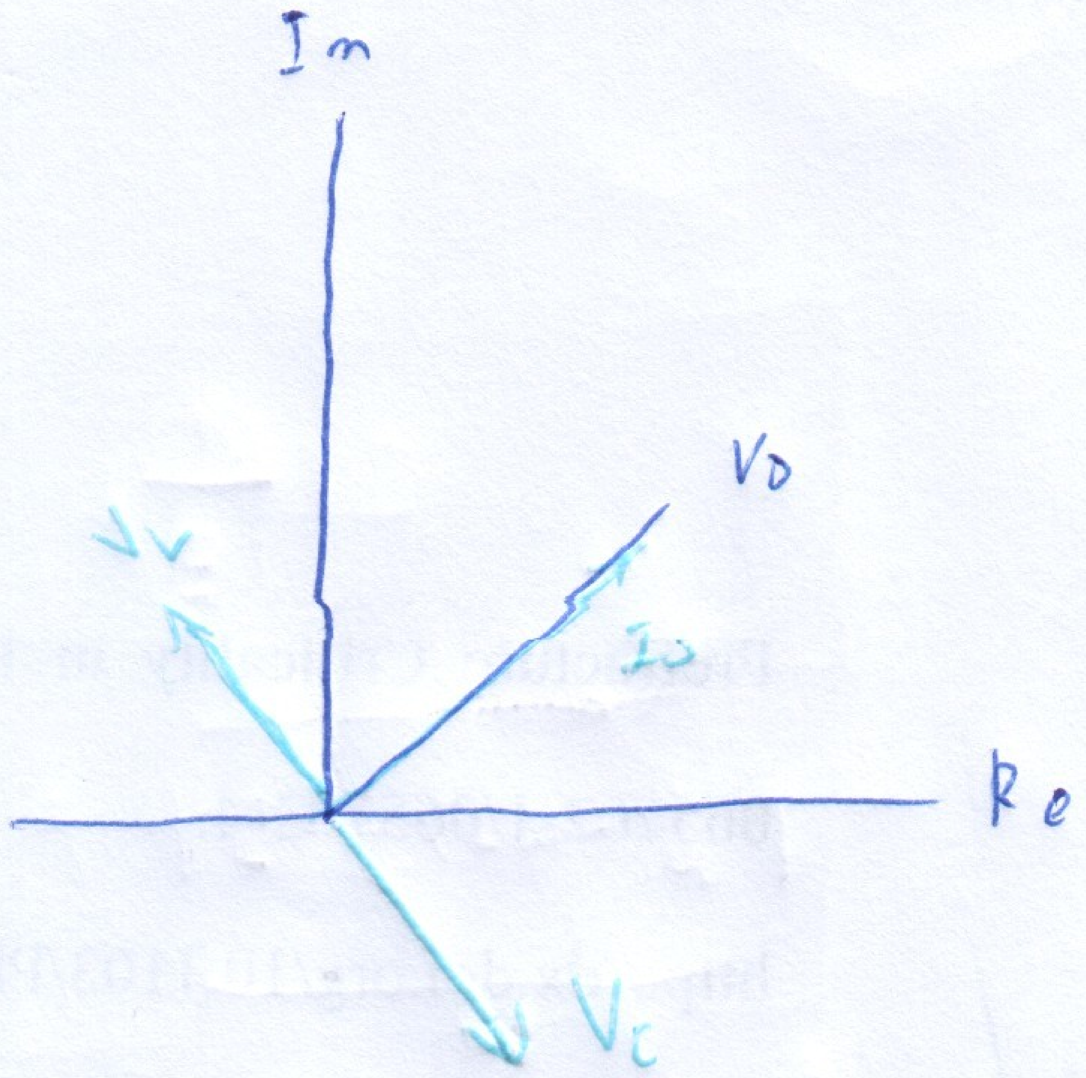
$Z_{min} = b$ ή $Z_{min} = R$

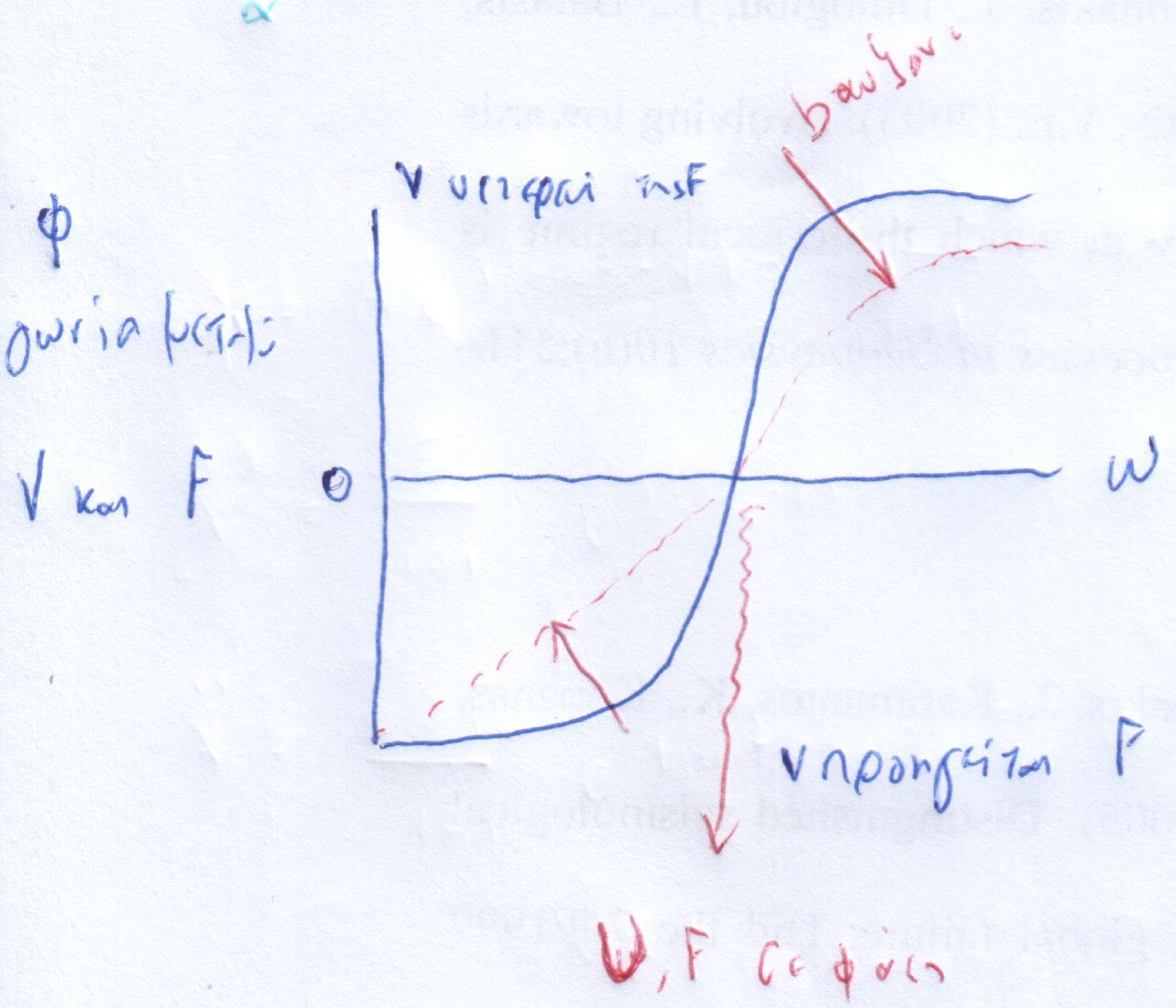
|Z|

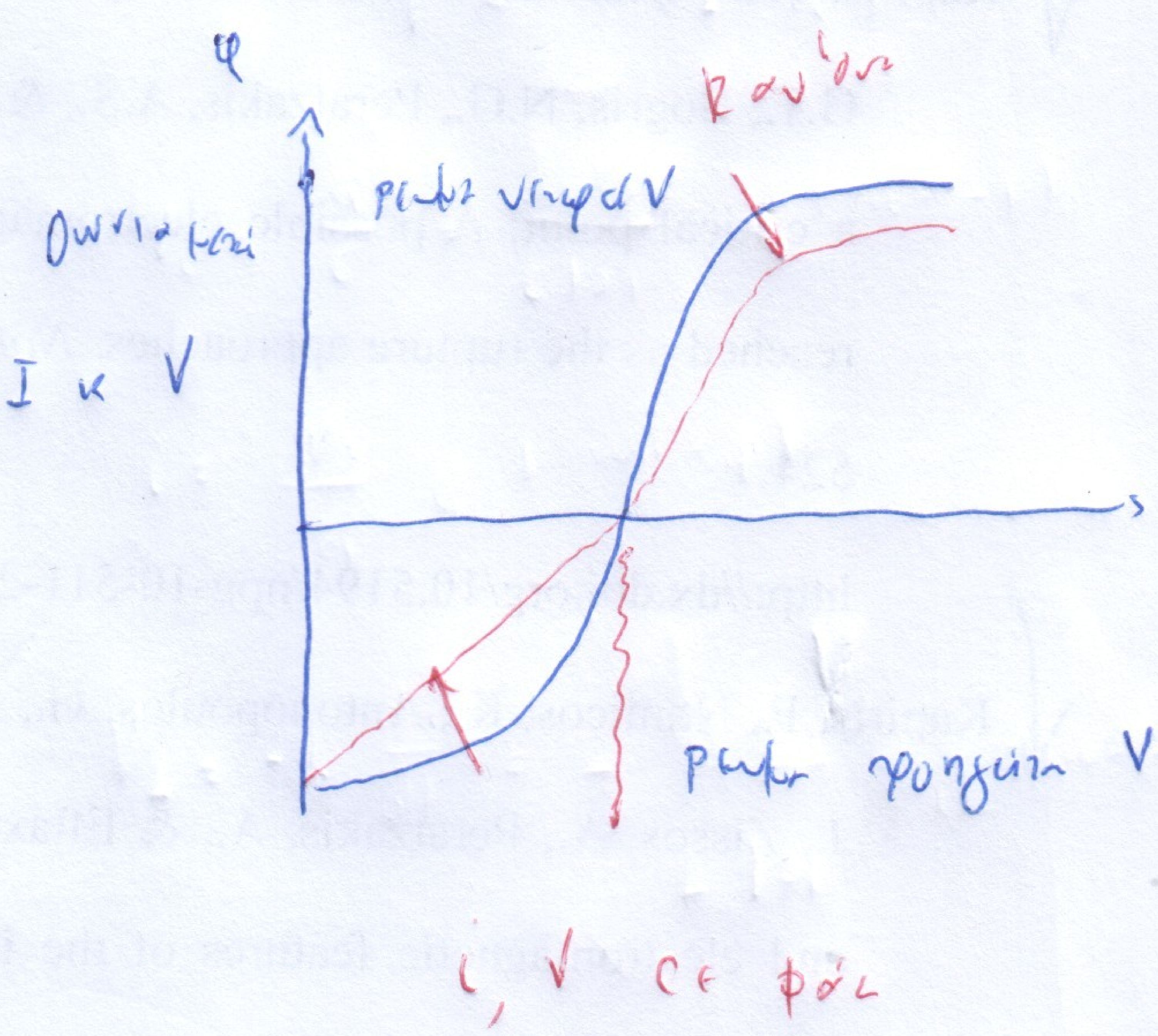


$$|Z| = \sqrt{b^2 + \left(m\omega - \frac{D}{\omega}\right)^2} \qquad |Z| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$









$$\boxed{v = \frac{F_0}{|Z|} e^{j(\omega t - \varphi)}}$$

$$\boxed{Z = b + j\left(m\omega - \frac{D}{\omega}\right)}$$

$$\boxed{\tan\varphi = \frac{m\omega - \frac{D}{\omega}}{b}}$$

$$i = \frac{V_0}{|Z|} e^{j(\omega t - \varphi)}$$

$$Z = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

$$\tan\varphi = \frac{\omega L - \frac{1}{\omega C}}{R} = \frac{Z_L - Z_C}{R}$$

$$\omega_0: \quad m\omega_0 = \frac{D}{\omega_0} \Rightarrow$$

$$\Rightarrow \boxed{D = m\omega_0^2}$$

$$\frac{1}{\omega_0 C} = \omega_0 L$$

$$\Rightarrow \boxed{\omega_0^2 = \frac{1}{LC}}$$

ΣΥΝΤΟΝΙΣΜΟΣ ΑΠΟΡΡΟΦΗΣΗ RLC

Στιγμιαία Ισχύς: $P(t) = V(t) I(t) = Re(V_0 e^{j\omega t}) Re(I_0 e^{j(\omega t - \phi)})$

$$\Rightarrow P(t) = V_0 \cos\omega t \cdot I_0 \cos(\omega t - \phi)$$

$$\Rightarrow \boxed{P(t) = V_0 I_0 \cos\omega t \cos(\omega t - \phi)}$$

$$\cos(a - b) = \cos a \cos b + \sin a \sin b$$

$$\Rightarrow P(t) = V_0 I_0 \cos\omega t \left(\cos\omega t \cos\phi + \sin\omega t \sin\phi\right)$$

$$\Rightarrow P(t) = V_0 I_0 \left(\cos^2\omega t \cos\phi + \cos\omega t \sin\omega t \sin\phi\right)$$

$$\sin 2\omega t = 2 \sin\omega t \cos\omega t$$

$$\Rightarrow P(t) = V_0 I_0 \left(\cos^2\omega t \cos\phi + \sin 2\omega t \sin\phi\right)$$

$$\Rightarrow \boxed{P(t) = \left(V_0 I_0 \cos\phi\right) \cos^2\omega t + \left(V_0 I_0 \sin\phi\right) \sin 2\omega t}$$

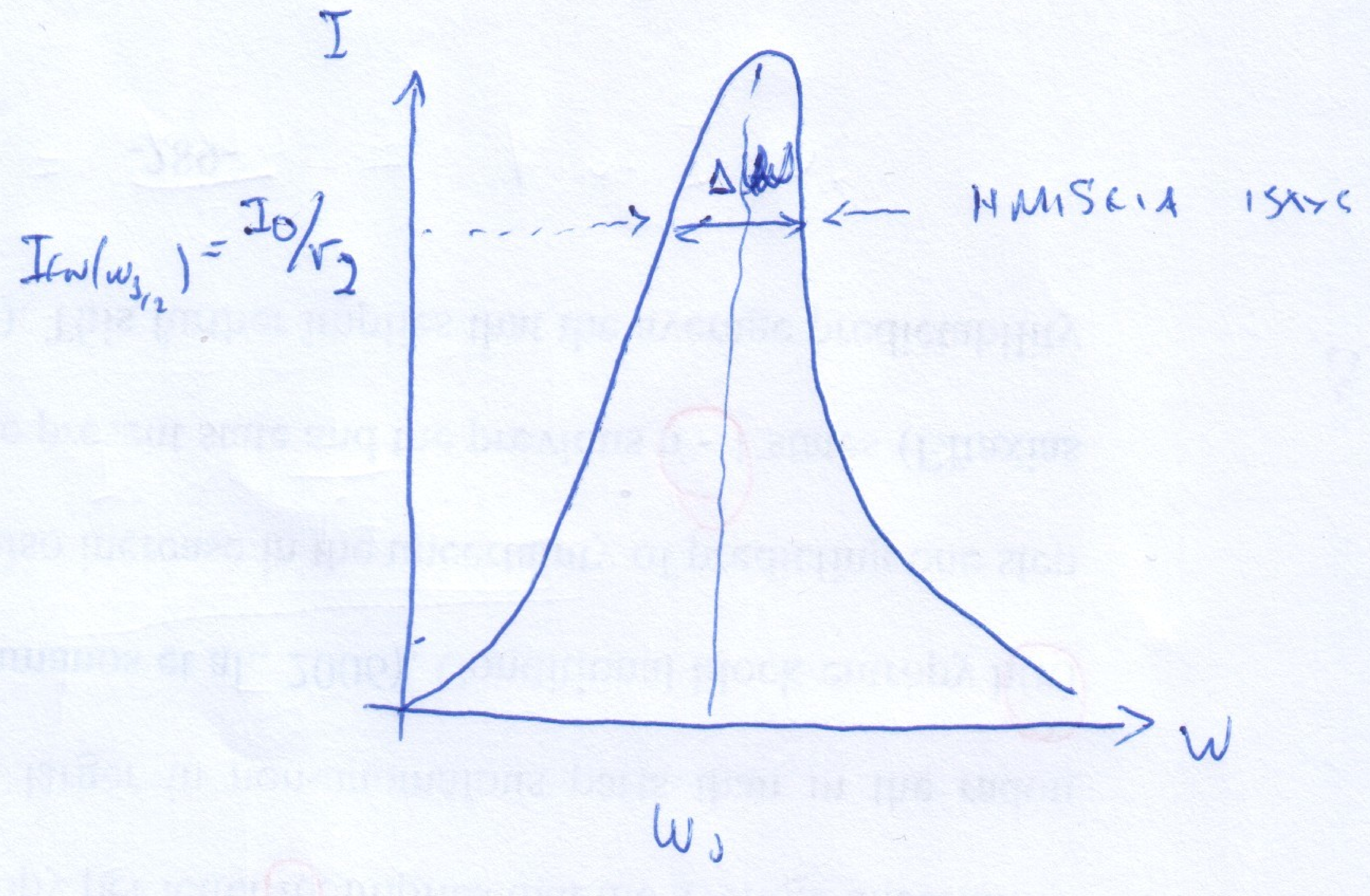
ΜΕΣΗ ΙΣΧΥΣ: $\langle P(t) \rangle = \frac{1}{T}\int_0^T P(t)\,dt = V_0 I_0 \frac{1}{T}\int_0^T \cos^2\omega t\,dt + \cancel{V_0 I_0 \sin\phi \frac{1}{T}\int_0^T \sin 2\omega t\,dt}^{\,0}$

$$= V_0 I_0 \frac{1}{2}\cos\phi \int_0^T \left(1 - \frac{1}{2}\cos 2\omega t\right) dt = V_0 I_0 \frac{1}{2}\cos\phi \int_0^T 1\,dt - \cancel{\frac{1}{2}\int \cos 2\omega t\,dt}^{\,0}$$

$$\Rightarrow \boxed{\langle P(t) \rangle = \frac{V_0 I_0}{2}\cos\phi} \Leftrightarrow \boxed{\langle P(t) \rangle = I_{\varepsilon\nu} V_{\varepsilon\nu} \cos\phi}$$

ΣΥΝΤΕΛΕΣΤΗΣ ΠΟΙΟΤΗΤΑΣ

$$Q = \frac{\omega_0}{\Delta\omega_{\text{ημίσειας ισχύος}}}$$



Μέγιστη ισχύς

$$P_{max} = \langle p(t) \rangle_{συντ} = \frac{V_0 \cdot I_0}{2} = V_{εν} \, I_{εν} = \frac{V_{εν}^2}{R} = \frac{V_0^2}{2R}$$

$$\Rightarrow \boxed{P_{max} = \frac{V_0^2}{2R} = \frac{1}{2} I_0^2 \cdot R}$$

Ημίσεια ισχύς

$$P_{1/2} = \frac{1}{2} P_{max} \Rightarrow P_{1/2} = \frac{1}{2} I_0^2 \cdot R$$

$$\Rightarrow I^2_{\substack{εν \\ \text{ημίσεια ισχύς}}} R = \frac{1}{2} I_0^2 R$$

$$\Rightarrow I^2_{\substack{εν \\ \text{ημίσεια ισχύς}}} = \frac{I_0^2}{2} \Rightarrow \boxed{I_{εν}(\omega_{1/2}) = \frac{I_0}{\sqrt{2}}}$$

Αλλά

$$I_{εν} = \frac{V_{εν}}{|Z|} = \frac{V_{εν}}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \Rightarrow$$

$$\Rightarrow \boxed{I_{εν}(\omega_{1/2}) = \frac{V_{εν}}{\sqrt{R^2 + \left(\omega_{1/2} L - \frac{1}{\omega_{1/2} C}\right)}}}$$

Επειδή

$$\boxed{I_{εν}(w_0) = \frac{V_{εν}}{R}} \quad κ' \quad \boxed{I_0 = \frac{V_0}{R}}$$

Αφού

$$I_{εν}(w_{1/2}) = \frac{I_0}{\sqrt{2}} = \frac{V_0}{R\sqrt{2}}$$

Δηλαδή

$$\frac{\cancel{V_{εν}}}{\sqrt{R^2 + \left(w_{1/2} L - \frac{1}{w_{1/2} C}\right)^2}} = \frac{1}{\sqrt{2}} \frac{\cancel{V_{εν}}}{R} \Rightarrow$$

$$\Rightarrow \sqrt{2} R = \sqrt{R^2 + \left(w_{1/2} L - \frac{1}{w_{1/2} C}\right)^2}$$

$$\Rightarrow 2R^2 = R^2 + \left(w_{1/2} L - \frac{1}{w_{1/2} C}\right)^2$$

$$\Rightarrow R^2 = \left(w_{1/2} L - \frac{1}{w_{1/2} C}\right)^2$$

$$\Rightarrow R = \left(w_{1/2} L - \frac{1}{w_{1/2} C}\right) \quad ή \quad R = -\left(w_{1/2} L - \frac{1}{w_{1/2} C}\right)$$

$$\Rightarrow \boxed{w_1 = \frac{-R + \sqrt{R^2 - 4\frac{L}{C}}}{2L} \quad \Big| \quad w_2 = \frac{+R + \sqrt{R^2 - 4\frac{L}{C}}}{2L}}$$

$$Q = \frac{\omega_0}{\Delta\omega_{1/2}} = \frac{\omega_0}{\omega_2 - \omega_1}$$

Αλλά $\omega_2 - \omega_1 = \frac{2R}{2L} = \frac{R}{L}$

$$Q = \frac{\omega_0}{R/L} \Rightarrow \boxed{Q = \frac{\omega_0 L}{R}}$$

Όμως

$$\omega_0 L = \frac{1}{\omega_0 C}$$

οπότε

$$\boxed{Q = \frac{1}{\omega_0 C R}}$$

ΓΕΝΙΚΑ

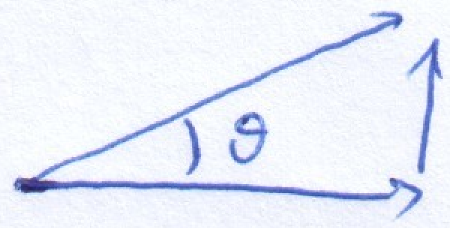
$$Q = 2\pi \frac{\text{ΜΕΓΙΣΤΗ ΑΠΟΘΗΚΕΥΜΕΝΗ ΕΝΕΡΓΕΙΑ}}{\text{ΕΝΕΡΓΕΙΑ ΠΟΥ ΚΑΤΑΝΑΛΩΝΕΤΑΙ ΣΕ 1 T}}$$

$$Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 C R}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$\boxed{Q = \frac{1}{R}\sqrt{\frac{L}{C}}}$$

ΣΥΝΤΟΝΙΣΜΟΣ ΣΥΝΑΡΤΗΣΗ ΣΥΧΝΟΤΗΤΑΣ ΠΟΙΟΤΗΤΑΣ



$Z_L - Z_C \qquad \tan\theta = \dfrac{[Z_L - Z_C]}{R}$

$$\Rightarrow \boxed{\tan\theta = \frac{\omega L - \frac{1}{\omega C}}{R} = \frac{\omega}{\omega_0} Q \left(1 - \frac{\omega_0^2}{\omega^2}\right)}$$

Vcl

$$I_0(\omega) = \frac{V_0}{|Z|} = \frac{V_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

$$= \frac{V_0/R}{\sqrt{1 + \left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)^2}}$$

$$\Rightarrow \boxed{I_0(\omega) = \frac{V_0/R}{\sqrt{1 + \frac{\omega^2}{\omega_0^2} Q^2 \left(1 - \frac{\omega_0^2}{\omega^2}\right)}}}$$

ΣΤΟ ΣΥΝΤΟΝΙΣΜΟ $\rightarrow \cos\phi = \cos 0 = 1$ οπότε

$$\boxed{\langle P(t)\rangle_{συντ} = \frac{V_0 I_0}{2} = V_{εν} I_{εν} = P_{max}}$$

Απόδειξη ότι $V_{εν} = \frac{V_0}{\sqrt{2}}$, $I_{εν} = \frac{I_0}{\sqrt{2}}$

$$V_{εν} = V_{rms} = \sqrt{\frac{1}{T}\int_0^T V^2 dt} = \sqrt{\frac{1}{T}\int_0^T V_0^2 \cos^2\omega t \, dt} =$$

$$= \sqrt{\frac{V_0^2}{T}\int_0^T \frac{1}{2}(1 - \cos 2\omega t)\, dt}$$

$$= \sqrt{\frac{V_0^2}{T}\left[\int_0^T \frac{1}{2} dt \cancel{+ \int_0^T \cos 2\omega t \, dt}\right]}$$

$$= \sqrt{\frac{V_0^2}{T}\frac{1}{2}T} \Rightarrow V_{εν} = \sqrt{\frac{V_0^2}{2}} \Rightarrow \boxed{V_{εν} = \frac{V_0}{\sqrt{2}}}$$

$$\langle P(t)\rangle_{ΣΥΝ} = V_{εν} \cdot I_{εν}^{ΣΥΝ}$$

$$I_{εν} = \frac{V_{εν}}{Z} \Rightarrow I_{εν}^{ΣΥΝΤ} = \frac{V_{εν}}{R}$$

$$\boxed{P_{max} = \langle P(t)\rangle = \frac{V_{εν}^2}{R} = I_{εν}^2 R}$$

$$P_{max} = \frac{V_0 I_0}{2} = \frac{V_0^2}{2R} = \frac{I_0^2}{2}R = I_{εν}^2 R$$