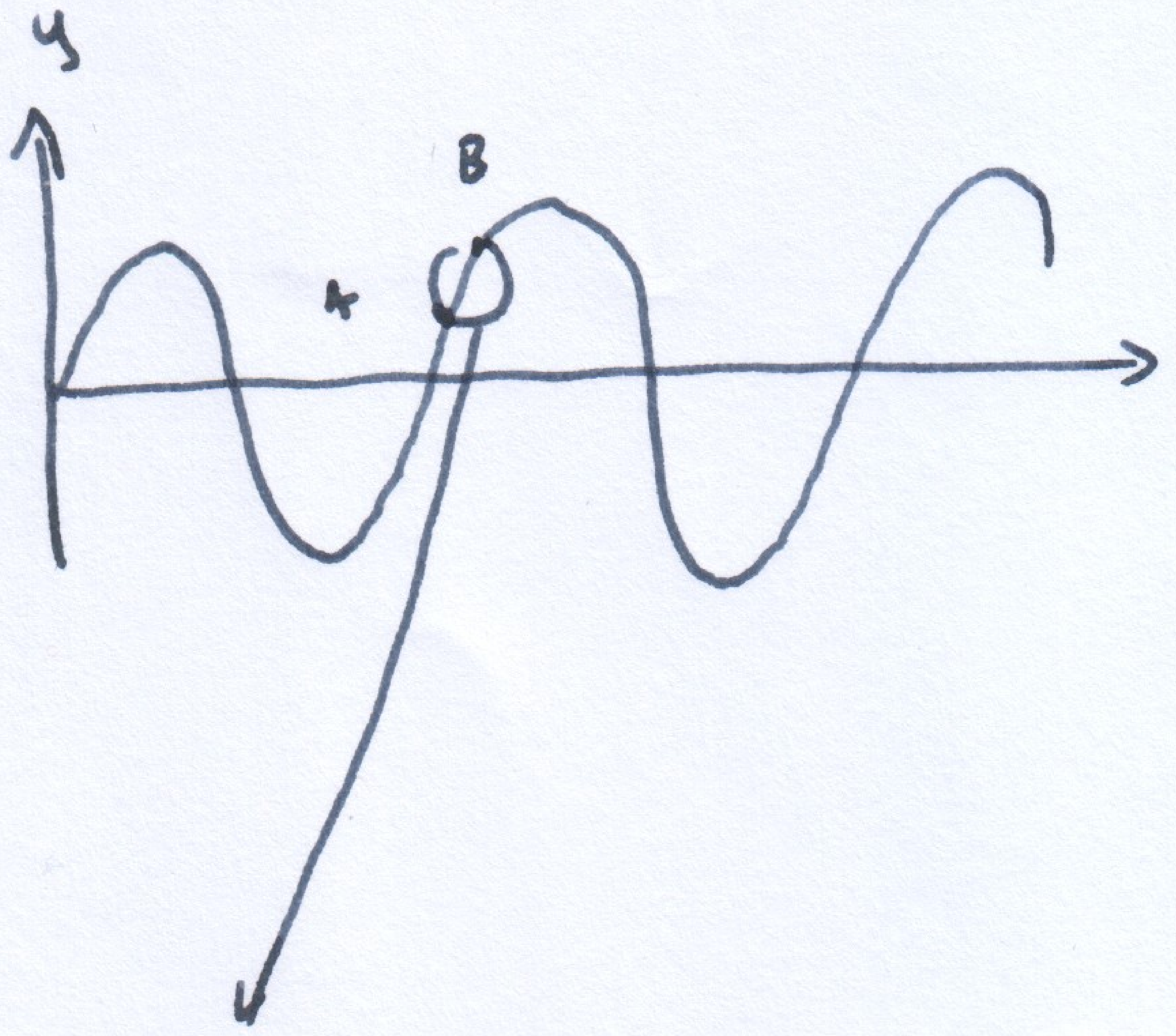
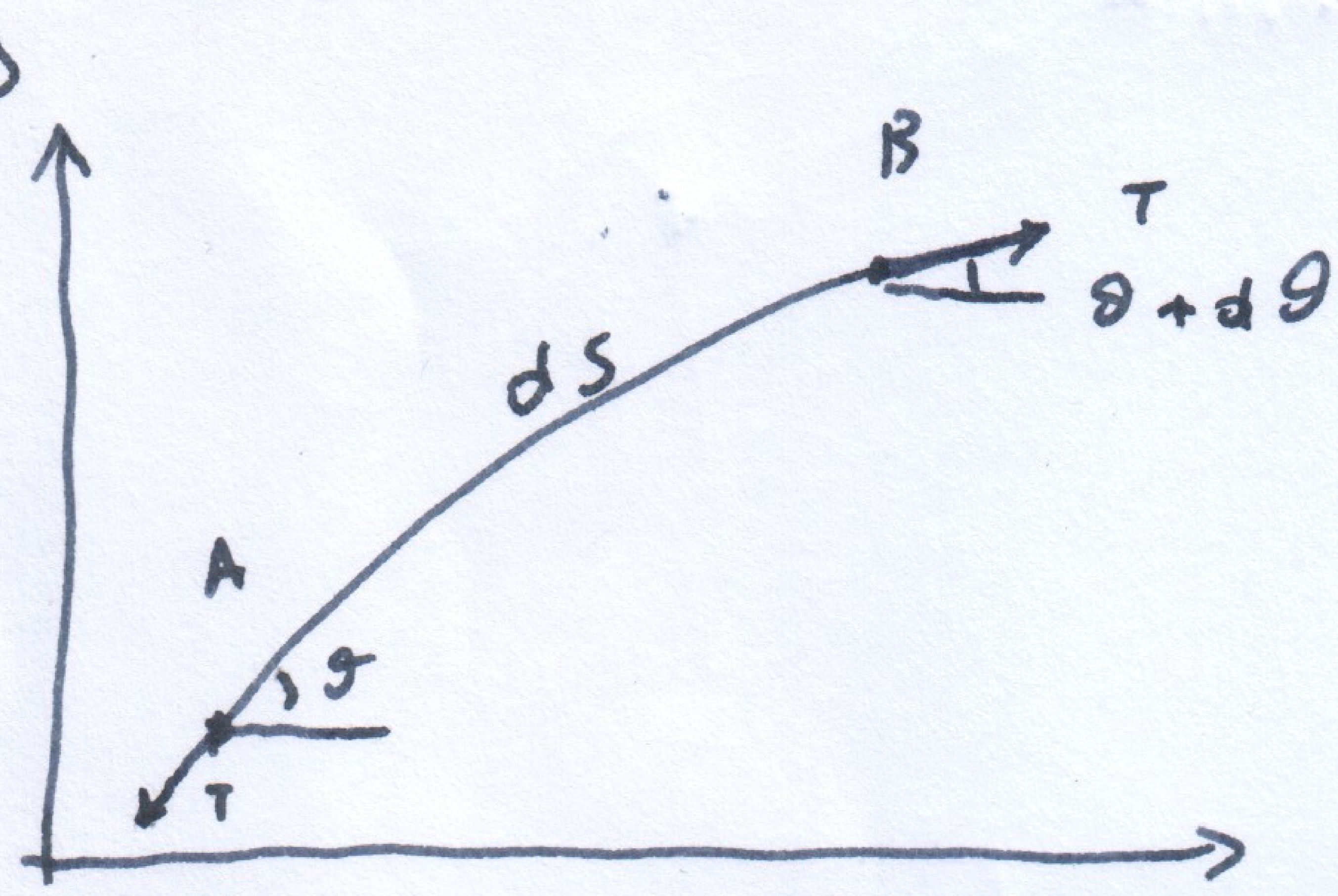
ΚΥΜΑΤΑ





x άξονας: $T\cos\theta = T\cos(\theta + d\theta)$ μη κίνηση στον x

$\Rightarrow T\cos\theta = T\cos\theta\cos d\theta - T\cancel{\sin\theta\sin d\theta}^{\,0}$

$\cos d\theta \simeq 1$

$$\Longrightarrow \boxed{T\cos\theta = T\cos\theta} \checkmark$$

Y-άξονας: $\Sigma F_y = T\sin(\theta + d\theta) - T\sin\theta$

$$m\frac{\partial^2 y}{\partial t^2} = T\left[\tan(\theta + d\theta) - \tan\theta\right] \quad (\theta \ll \Rightarrow \sin\theta \simeq \tan\theta)$$

$$\Rightarrow m\frac{\partial^2 y}{\partial t^2} = T\left[\left(\frac{\partial y}{\partial x}\right)_{x+dx} - \left(\frac{\partial y}{\partial x}\right)_x\right] \quad \left(\text{αφού } \tan\theta = \frac{dy}{dx}\right)$$

$$\Rightarrow m\frac{\partial^2 y}{\partial t^2} = T\frac{\partial^2 y}{\partial x^2}\cdot dx$$

$m = p \cdot dx$

$$\Longrightarrow p\cdot\cancel{dx}\frac{\partial^2 y}{\partial t^2} = T\cancel{dx}\frac{\partial^2 y}{\partial x^2}$$

$$\Rightarrow \boxed{\frac{\partial^2 y}{\partial x^2} = \frac{p}{T}\frac{\partial^2 y}{\partial t^2}} \Leftrightarrow \boxed{\frac{\partial^2 y}{\partial x^2} = \frac{1}{\upsilon^2}\frac{\partial^2 y}{\partial t^2}}$$

ΣΗΜΑΝΤΙΚΟ: $v = \sqrt{\frac{T}{\rho}} = \sqrt{\frac{\text{ελαστικότητα}}{\text{αδράνεια}}}$

$\boxed{v = \frac{dx}{dt}}$

$v \equiv$ κυματική ταχύτητα

$v \equiv$ φασική ταχύτητα =

= η ταχύτητα με την οποία οδεύουν επίπεδα ίδιας φάσης

Λύση Κυματικής Εξίσωσης

ΚΑΘΕ ΣΥΝΑΡΤΗΣΗ $f(vt \pm x)$ ΑΠΟΤΕΛΕΙ ΛΥΣΗ

Πράγματι για $f(vt-x)$

$$\left.\begin{aligned} \frac{\partial y}{\partial x} &= -f(vt-x) \Rightarrow \frac{\partial^2 y}{\partial x^2} = f(vt-x) \\ \frac{\partial y}{\partial t} &= v\, f(vt-x) \Rightarrow \frac{\partial^2 y}{\partial t^2} = v^2 f(vt-x) \end{aligned}\right\} \Rightarrow \boxed{\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}}$$

ΑΡΜΟΝΙΚΑ ΚΥΜΑΤΑ

$$f = \begin{cases} \sin \\ \cos \end{cases} \qquad \boxed{y = \alpha \sin(\omega t - kx)}$$

Ισοδύναμες εκφράσεις

$$y = \alpha \sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right) \qquad y = \alpha \sin \frac{2\pi}{T}\left(t - \frac{x}{v}\right)$$

$$y = \alpha \sin \frac{2\pi}{\lambda}(vt - x)$$

0	φ	2π
0	t	T
0	x	λ

$\varphi = 2\pi \frac{t}{T}$

$\varphi = 2\pi \frac{x}{\lambda}$

$$\boxed{v = \frac{dx}{dt}} \Rightarrow v = \frac{x}{t} = \frac{\lambda}{T} \Rightarrow$$

$$\boxed{\lambda = vT} \quad \text{ή} \quad \boxed{\lambda = v/f}$$

$y = \alpha \sin(\omega t - kx)$ $\qquad$ $y = \alpha \cos(\omega t - kx)$

$$\boxed{y = \alpha e^{j(\omega t - kx)}} \quad \rightsquigarrow$$

$$\boxed{y = \alpha e^{j(\omega t + kx)}} \quad \leftarrow$$

ΤΑΧΥΤΗΤΑ ΚΥΜΑΤΟΣ: $\boxed{\upsilon = \dfrac{\partial x}{\partial t}}$

$$y = \alpha \sin(\omega t - kx) \Rightarrow \frac{\partial y}{\partial t} = \omega \alpha \cos(\omega t - kx)$$

Επίσης $\dfrac{\partial y}{\partial x} = -k\alpha \cos(\omega t - kx)$

ΑΡΑ

$$-\frac{1}{k}\frac{\partial y}{\partial x} = \alpha \cos(\omega t - kx) = \frac{1}{\omega}\frac{\partial y}{\partial t}$$

$$\Rightarrow \boxed{\frac{\partial y}{\partial t} = -\frac{\omega}{k}\frac{\partial y}{\partial x}}$$

$$\Rightarrow \boxed{\upsilon_T = -\frac{\omega}{k}\frac{\partial y}{\partial x}}$$

Επειδή $\dfrac{\partial y}{\partial t} = \dfrac{\partial y}{\partial x}\dfrac{\partial x}{\partial t} \Rightarrow -\dfrac{\omega}{k}\dfrac{\partial y}{\partial x} = \dfrac{\partial y}{\partial x}\dfrac{\partial x}{\partial t} \Rightarrow \boxed{\dfrac{\partial x}{\partial t} = -\dfrac{\omega}{k}}$.

ΤΑΧΥΤΗΤΑ ΚΥΜΑΤΟΣ : $v = \frac{\partial x}{\partial t} = -\frac{\omega}{k}$

Ταχύτητα ταλάντωσης σημείου : $v_T = \frac{\partial y}{\partial t} = -\frac{\omega}{k}\frac{\partial y}{\partial x}$

Αν $\omega = \omega(k)$ [διασπορά] τότε $\boxed{v_g = \frac{d\omega}{dk}}$

$$v_g = \frac{d\omega}{dk} \Rightarrow v_g = \frac{d}{dk}(kv) \Rightarrow v_g = v\frac{dk}{dk} + k\frac{dv}{dk} \Rightarrow \boxed{v_g = v + k\frac{dv}{dk}}$$

ΣΥΝΘΕΤΗ ΑΝΤΙΣΤΑΣΗ ΧΟΡΔΗΣ

$$\boxed{Z = \frac{F}{v}}$$

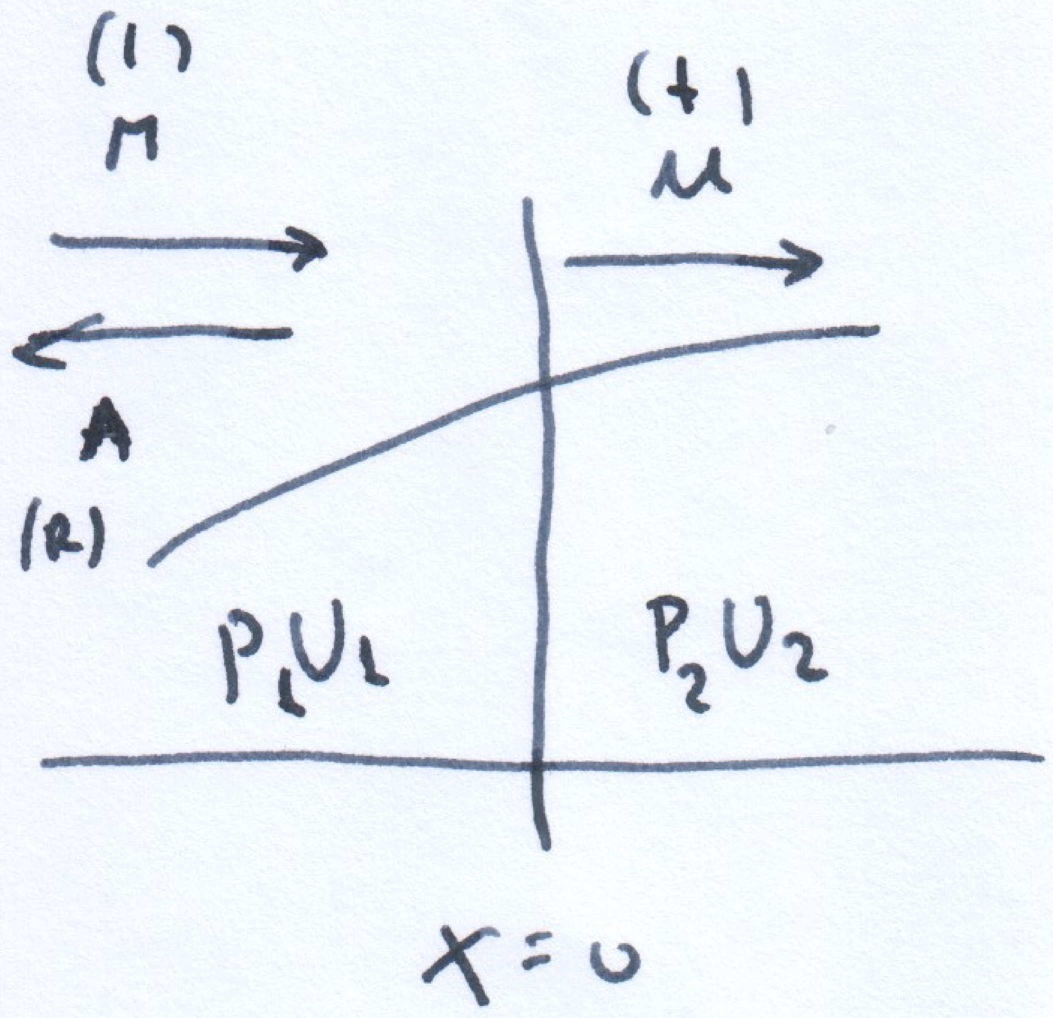
$F = F_0 e^{i\omega t}$ η (μιγαδική) εξωτερική δύναμη στο $x=0$ ώστε να αποκατασταθεί κύμα.

Πχ για τη χορδή $F = T \Rightarrow Z = \frac{T}{v}$

αλλά $\frac{T}{\rho} = v^2 \Rightarrow T = v^2 \cdot \rho$ $\Bigg\}$ $Z = \frac{v^{\cancel{2}}\rho}{\cancel{v}}$

$$\Rightarrow Z = v \cdot \rho \Rightarrow \boxed{Z = \rho v}$$

ΑΝΑΚΛΑΣΗ ΚΑΙ ΜΕΤΑΔΟΣΗ ΚΥΜΑΤΩΝ ΣΕ ΣΥΝΟΡΟ



Π: προσπίπτον (INCIDENT)

Α: ανακλώμενο (REFLECTED)

Μ: μεταδιδόμενο (TRANS)

<u>Προσαρμογή</u>:
$$y_i + y_r = y_t \Rightarrow$$

$$\Rightarrow A_1 e^{j(\omega t - k_1 x)} + B_1 e^{j(\omega t + k_1 x)} = A_2 e^{j(\omega t - k_2 x)}$$

$$\xRightarrow{x=0} A_1 e^{j\omega t} + B_1 e^{j\omega t} = A_2 e^{j \omega t}$$

$$\xRightarrow{x=0} \boxed{A_1 + B_1 = A_2}$$

$T \sin\theta \simeq T \tan\theta = T\left(\frac{\partial y}{\partial x}\right)$ συνεχής στο $x=0$

$$\Rightarrow T \frac{\partial}{\partial x}(y_i + y_r) = T \frac{\partial}{\partial y} y_t$$

$$\xRightarrow{x=0} T\left(\frac{\partial}{\partial x}\left(A_1 e^{j(\omega t - k_1 x)} + B_1 e^{j(\omega t + k_1 x)}\right)\right)$$

$$= T \frac{\partial}{\partial x} A_2 e^{j(\omega t - k_2 x)}$$

$$\xrightarrow{x=0} T\left(-k_1 A_1 \cancel{e^{j\omega t}}\right) + T B_1 k_1 \cancel{e^{j\omega t}} = T\left(-k_2 A_2 \cancel{e^{j\omega t}}\right)$$

$$\Rightarrow \boxed{-k_1 T A_1 + k_1 B_1 T = -k_2 T A_2}$$

$$\lambda = v/f \Rightarrow \lambda = v \cdot T \Rightarrow \frac{2\pi}{\lambda} = \frac{2\pi}{v \cdot T} \Rightarrow \frac{2\pi}{\lambda} = \frac{1}{v}\,\frac{2\pi}{T} \Rightarrow$$

$$\Rightarrow k = \frac{1}{v}\,\omega \Rightarrow \boxed{k = \frac{\omega}{v}}$$

Άρα

$$-\frac{\omega_1}{v_1} T A_1 + \frac{\omega_1}{v_1} T B_1 = -\frac{\omega_2}{v_2} T A_2$$

$$\xrightarrow[1-A]{\omega_1=\omega_2=\omega} -\frac{\omega}{v_1} T A_1 + \frac{\omega}{v_1} T B_1 = -\frac{\omega}{v_2} T A_2$$

$$\Rightarrow -\omega \frac{T}{v_1} A_1 + \omega \frac{T}{v_1} B_1 = -\omega \frac{T}{v_2} A_2$$

$$\xrightarrow{Z = \frac{T}{v}} -\cancel{\omega} Z_1 A_1 + \cancel{\omega} Z_1 B_1 = -\cancel{\omega} Z_2 A_2$$

$$\Rightarrow Z_1 \cdot (A_1 - B_1) = Z_2 \cdot A_2$$

$$\Rightarrow \boxed{Z_1 (A_1 - B_1) = Z_2 A_2}$$

Συντελεστής Ανάκλασης $R = \dfrac{B_1}{A_1}$

Συντελεστής Μετάδοσης : $T = \dfrac{A_2}{A_1}$

$$\left.\begin{aligned} A_1 + B_1 &= A_2 \\ Z_1 (A_1 - B_1) &= Z_2 A_2 \end{aligned}\right\} \Rightarrow \boxed{\begin{aligned} B_1 &= \frac{Z_1 - Z_2}{Z_1 + Z_2} A_1 \\ A_2 &= \frac{2 Z_1}{Z_1 + Z_2} A_1 \end{aligned}}$$

$$\Longrightarrow \quad \boxed{R = \frac{Z_1 - Z_2}{Z_1 + Z_2}}$$

$$\boxed{T = \frac{2 Z_1}{Z_1 + Z_2}}$$

ΕΝΕΡΓΕΙΑ ΤΑΛΑΝΤΟΥΜΕΝΗΣ ΧΟΡΔΗΣ

Κινητική Ενέργεια στοιχείου dx : $\boxed{K = \frac{1}{2}\int_0^{\ell} \rho \cdot y^2 \cdot dx}$

$$dk = \frac{1}{2} dm\, y^2 = \frac{1}{2} \rho\, dx \cdot y^2 \Rightarrow \boxed{dk = \frac{1}{2}\rho\, dx\, y^2}$$

Δυναμική Ενέργεια χορδής : $\boxed{U = \frac{1}{2} T \int_0^{\ell} \left(\frac{\partial y}{\partial x}\right)^2 \cdot dx}$

$dU = dW_T$ (για έκταση ενός στοιχείου μήκους dx σε μήκος ds)

$$\left.\begin{aligned} &\Rightarrow dU = T\cdot(ds - dx) \\ &ds = \sqrt{dx^2 + dy^2} = \sqrt{1 + \left(\frac{\partial y}{\partial x}\right)^2 dx} \end{aligned}\right\} dU = T\left[dx\sqrt{1+\left(\frac{\partial y}{\partial x}\right)^2} - dx\right]$$

$$\Rightarrow dU = T dx\left[\sqrt{1+\left(\frac{\partial y}{\partial x}\right)^2} - 1\right] \Rightarrow$$

$$\Rightarrow dU \simeq T\,dx\left[\left(1+\left(\frac{\partial y}{\partial x}\right)^2\right)^{1/2} - 1\right]$$

$$\Rightarrow dU \simeq T\,dx\left[\cancel{1} + \frac{1}{2}\left(\frac{\partial y}{\partial x}\right)^2 \cancel{-1}\right]$$

$$\Rightarrow \boxed{dU = \frac{1}{2} T \left(\frac{\partial y}{\partial x}\right)^2 dx}$$

ΟΛΙΚΗ ΕΝΕΡΓΕΙΑ : $E = K + U$

$$E_{ολ} = \frac{1}{2} D A^2 \xRightarrow{D = dm\,\omega^2} E_{ολ} = \frac{1}{2} dm\,\omega^2 A^2 \Rightarrow \boxed{E_{ολ} = \frac{1}{2}\rho\omega^2 A^2 dx}$$

ΕΝΕΡΓΕΙΑ ΚΥΜΑΤΟΣ

$$E = K + U \Rightarrow E = \frac{1}{2} D A^2 \xRightarrow{D = m\omega^2} E = \frac{1}{2} m \omega^2 A^2$$

$$\xRightarrow[\chi o \rho \delta \eta]{1-D} \boxed{dE = \frac{1}{2} \rho \omega^2 A^2 dx}$$ αφού $\rho dx = dm$

Ισχύς κύματος

$$P = \frac{dE}{dt} \Rightarrow P = \frac{dE}{dx} \frac{dx}{dt} \Rightarrow P = \frac{dE}{dx} \cdot \upsilon$$

$$\Rightarrow \boxed{P = \frac{1}{2} \rho \omega^2 A^2 \cdot \upsilon} = \frac{\text{ενέργεια ανά μονάδα μήκους}}{\Delta t}$$

$\|$

ενέργεια που μεταφέρεται ανά μονάδα μήκους ανά μονάδα χρόνου

$$P = \frac{1}{2} \rho \upsilon \omega^2 A^2 \Longrightarrow$$

$$\Rightarrow \boxed{P = \frac{1}{2} Z \omega^2 A^2}$$

ΑΝΑΚΛΑΣΗ ΚΑΙ ΜΕΤΑΔΟΣΗ ΕΝΕΡΓΕΙΑΣ

$$E_{INC} = E_{TRANS} + E_{REFL}$$

$$\Rightarrow \frac{1}{2}\rho_1 \upsilon_1 \omega^2 A_1^2 = \frac{1}{2}\rho_2 \upsilon_2 \omega^2 A_2^2 + \frac{1}{2}\rho_1 \upsilon_1 \omega^2 B_1^2$$

$$\Rightarrow \boxed{\frac{1}{2} Z_1 \omega^2 A_1^2 = \frac{1}{2} Z_2 \omega^2 A_2^2 + \frac{1}{2} Z_1 \omega^2 B_1^2}$$

$$\frac{R_E}{I_E} = \frac{\frac{1}{2} Z_1 \omega^2 B_1^2}{\frac{1}{2} Z_1 \omega^2 A_1^2} \Rightarrow \frac{R_E}{I_E} = \frac{Z_1 B_1^2}{Z_1 A_1^2}$$

$$\frac{B_1}{A_1} = \frac{Z_1 - Z_2}{Z_1 + Z_2}$$

$$\Rightarrow$$

$$\boxed{\frac{R_E}{I_E} = \left(\frac{Z_1 - Z_2}{Z_1 + Z_2}\right)^2}$$

$$\frac{T_E}{I_E} = \frac{\frac{1}{2} Z_2 \omega^2 A_2^2}{\frac{1}{2} Z_1 \omega^2 A_1^2} \Rightarrow \boxed{\frac{T_E}{I_E} = \frac{4 Z_1 Z_2}{(Z_1 + Z_2)^2}}$$

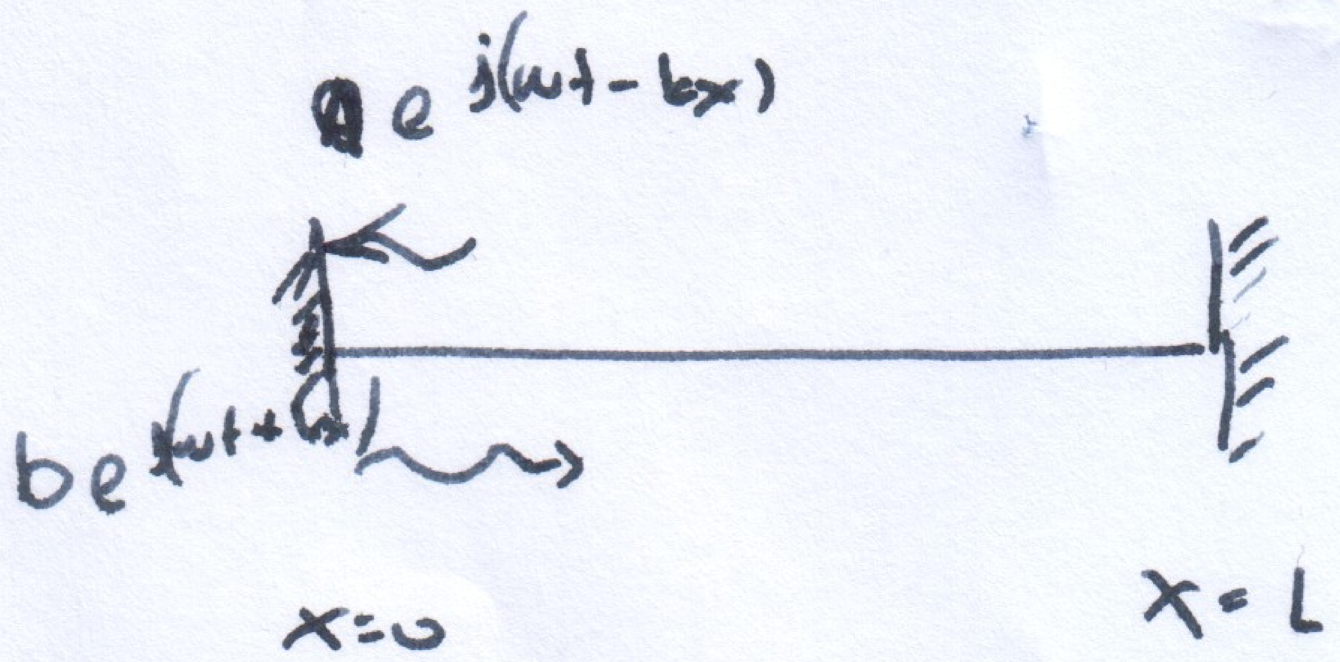
Αν $Z_1 = Z_2$ τότε

$$R_E = 0$$

$$T_E = I_E$$

ΔΕΝ ΑΝΑΚΛΑΤΑΙ ΚΑΘΟΛΟΥ ΕΝΕΡΓΕΙΑ.

ΣΤΑΣΙΜΑ ΚΥΜΑΤΑ ΣΕ ΧΟΡΔΗ



$$\boxed{y = \alpha e^{j(\omega t - kx)} + b e^{j(\omega t + kx)}}$$

Για x=0: $y_0 = \alpha e^{j\omega t} + b e^{j\omega t} \xrightarrow[y=0]{t=0} 0 = a e^{j\omega \cdot 0} + b e^{j\omega 0}$

$$\Rightarrow 0 = \alpha + b \Rightarrow \boxed{b = -\alpha}$$

Για x=L $y = 0$ (πάντοτε) $\Rightarrow$

$$\overset{b=-\alpha}{\Rightarrow} \alpha e^{j(\omega t - kL)} - \alpha e^{j(\omega t + kL)} = 0$$

$$\Rightarrow \alpha \left[e^{j(\omega t - kL)} - e^{j(\omega t + kL)}\right] = 0$$

$$\Rightarrow \alpha \left[e^{j\omega t} \cdot e^{j-kL} + e^{j\omega t} \cdot e^{jkL} \right] = 0$$

$$\Rightarrow \cancel{\alpha e^{j\omega t}} \left[e^{-jkL} - e^{jkL} \right] = 0$$

$$\left(e^{jx} = \cos x + j \sin x \right)$$

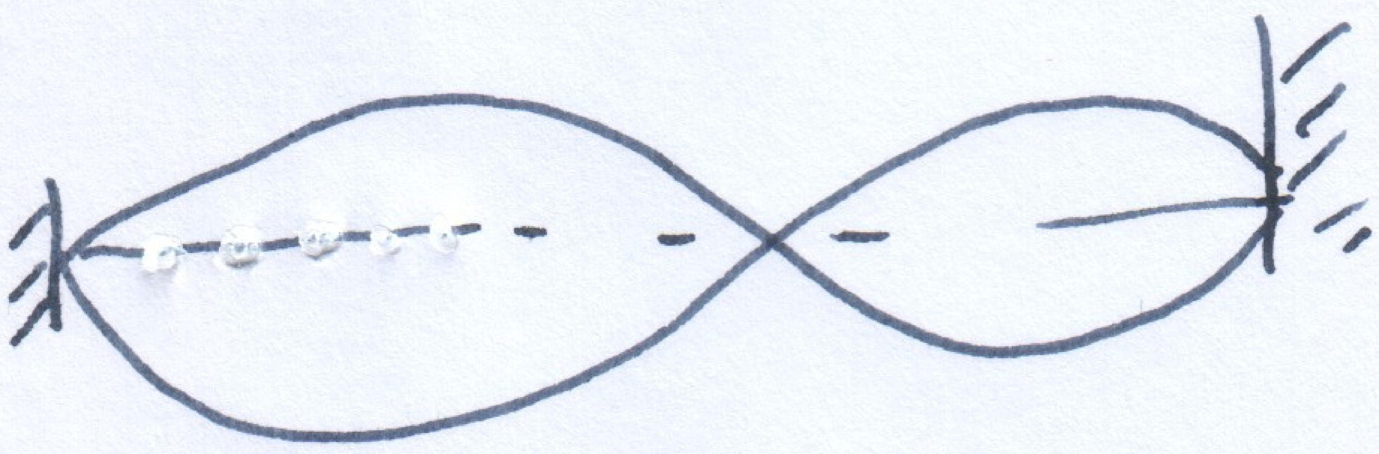
$$(-2j)\alpha \sin kL = 0 \Rightarrow (-j) 2\alpha \sin kL = 0$$

$$\Rightarrow kL = n \cdot \pi \Rightarrow \frac{2\cancel{\pi}}{\lambda_n} L = n\cancel{\pi} \Rightarrow \boxed{\lambda_n = \frac{2L}{n}}$$

$\lambda_0 = 2L$

$\Rightarrow L = \frac{\lambda_0}{2}$

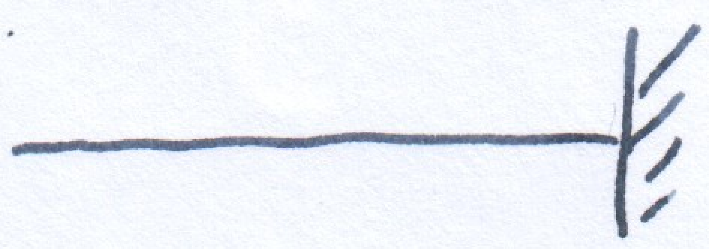
$\lambda_1 = \frac{2L}{2} = L$



$L = \lambda_1$

<u>ΑΝ ΤΟ ΕΝΑ ΑΚΡΟ ΤΗΣ ΧΟΡΔΗΣ ΕΙΝΑΙ ΕΛΕΥΘΕΡΟ $(x=0)$</u>

$$y = \alpha e^{j(\omega t - kx)} + b e^{j(\omega t + kx)}$$



$$y(x=L) = 0 \Rightarrow \alpha e^{j(\omega t - kL)} + b e^{j(\omega t + kL)} = 0$$

$$\Rightarrow \alpha e^{j\omega t} e^{-jkL} + b e^{j\omega t} e^{jkL} = 0$$ (για όλα τα t).

Στο x=L συνολι το κύμα άπειρη αντίσταση άρα

$$R = I \Rightarrow \alpha = b$$

Άρα για όλα τα t

$$\alpha e^{j\omega t} e^{-jkL} + \alpha e^{j\omega t} e^{jkL} = 0$$

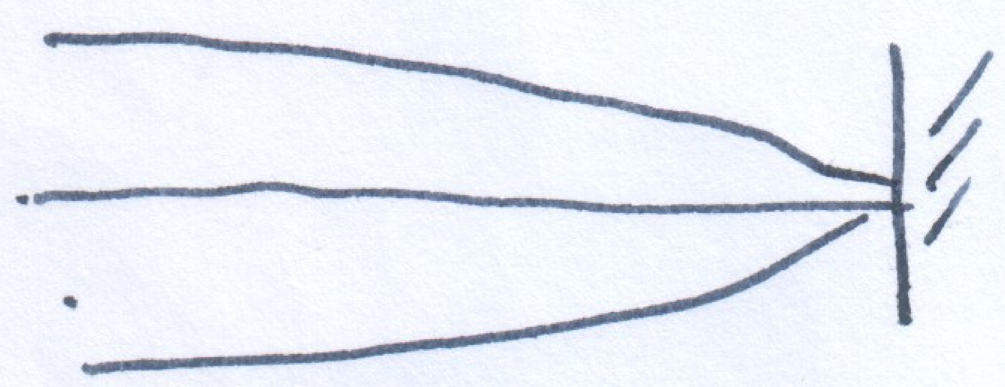
$$\cancel{\alpha e^{j\omega t}}\left(e^{-jkL} + e^{+jkL}\right) = 0$$

$$\rightarrow 2\alpha \cos k_n L = 0$$

$$\Rightarrow k_n L = (2n+1)\frac{\pi}{2} \Rightarrow \frac{2\pi}{\lambda_n} L = (2n+1)\frac{\pi}{2}$$

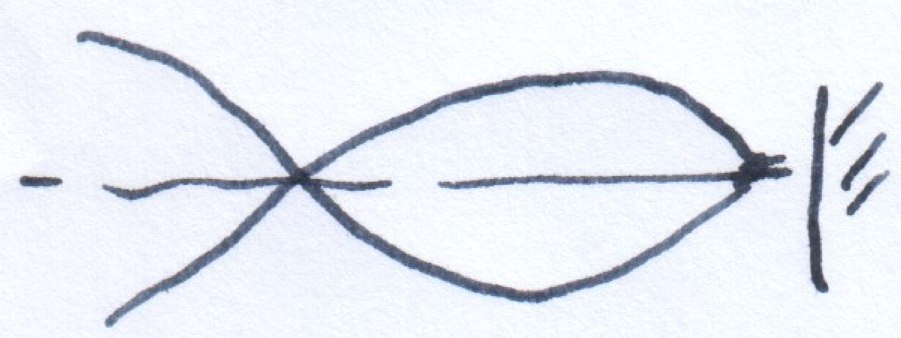
$$\Rightarrow \frac{4L}{2n+1} = \lambda_n \Rightarrow \boxed{\lambda_n = \frac{4L}{2n+1}}$$

$\lambda_0 = 4L$



$L = \lambda_0$

$\lambda_1 = \frac{4L}{3}$



$L = \frac{3\lambda_1}{4}$

ΠΛΑΤΟΣ ΣΤΑΣΙΜΩΝ 2α