# ΕΞΙΣΩΣΕΙΣ MAXWELL (B, E)

(1) ΝΟΜΟΣ GAUSS

$$\mathrm{div}\,E = \frac{\rho}{\varepsilon_0} \qquad \oint_S \vec{E}\cdot d\vec{S} = \frac{q_{in}}{\varepsilon_0}$$

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_0}$$

(4) ΝΟΜΟΣ AMPERE

$$\mathrm{curl}\,\vec{B} = \mu_0 \vec{J}_{FREE} + \mu_0\varepsilon_0 \frac{\partial \vec{E}}{\partial t} \qquad \oint_C \vec{B}\cdot d\vec{l} = \mu_0 i$$

$$\vec{\nabla}\times\vec{B} = \mu_0 \vec{J}_{FREE} + \mu_0\varepsilon_0 \frac{\partial \vec{E}}{\partial t} \qquad \oint_C \vec{B}\cdot d\vec{l} = \int_{S(C)} \vec{J}\cdot d\vec{S}$$

(3) ΝΟΜΟΣ FARADAY-HENRY:

$$\mathrm{curl}\,\vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \oint_C \vec{E}\cdot d\vec{l} = -\frac{d}{dt}\oint_{S(C)} \vec{B}\cdot d\vec{S}$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad E_{EH} = -\frac{d\Phi_B}{dt}$$

(2) ΑΝΥΠΑΡΞΙΑ ΜΟΝΟΠΟΛΩΝ

$$\mathrm{div}\,\vec{B} = 0 \qquad \oint_S \vec{B}\cdot d\vec{S} = 0$$

$$\vec{\nabla}\cdot\vec{B} = 0$$

## ΣΧΕΣΗ B − H / D − E

$\vec{B} = \mu \vec{H}$ : $\mu$ μαγνητική διαπερατότητα υλικού

$[\mu] = \frac{\text{Tesla}\cdot\text{m}}{\text{A}}$

$[H] = \frac{\text{A}}{\text{m}}$ (μαγνητικό πεδίο)

$[B] = \text{Tesla}$

$\vec{D} = \varepsilon \vec{E}$ : διηλεκτρική σταθερά υλικού

$[\varepsilon] = \frac{\text{Farad}}{\text{m}}$

$[D] = \frac{\text{Coulomb}}{\text{m}^2}$

$[E] = \text{N}/\text{C} = \text{V}/\text{m}$

# ΕΞΙΣΩΣΕΙΣ MAXWELL (H, D)

① $\text{div}\,\vec{D} = \rho$ $\qquad\qquad$ $\oint_S \vec{D}\cdot d\vec{S} = \dfrac{Q_{IN}}{\varepsilon\cdot\varepsilon_0}$

$\vec{\nabla}\cdot\vec{D} = \rho$

② $\text{div}\,\vec{H} = 0$ $\qquad\qquad$ $\oint \vec{H}\cdot d\vec{S} = 0$

$\vec{\nabla}\cdot\vec{H} = 0$

③ $\text{curl}\,\vec{E} = -\mu \dfrac{\partial \vec{H}}{\partial t}$ $\qquad\qquad$ $\oint_l \vec{E}\cdot dl = -\mu \dfrac{d}{dt} \oint_{S(l)} \vec{H}\cdot d\vec{S}$

$\vec{\nabla}\times\vec{E} = -\mu \dfrac{\partial \vec{H}}{\partial t}$

④ $\text{curl}\,\vec{H} = \varepsilon \dfrac{\partial \vec{E}}{\partial t} + \vec{J}_{free}$ $\qquad\qquad$ $\oint \vec{H}\cdot dl = \mu\mu_0 i$

$\vec{\nabla}\times\vec{H} = \varepsilon \dfrac{\partial \vec{E}}{\partial t} + \vec{J}_{free}$

$\text{curl}\,\vec{H} = \dfrac{\partial \vec{D}}{\partial t} + \vec{J}_{FREE}$

$\vec{\nabla}\times\vec{H} = \dfrac{\partial \vec{D}}{\partial t} + J_{FREE}$

## ΣΧΟΛΙΑ ΥΠΟΛΟΓΙΣΜΩΝ

$$\vec{\nabla}\times\vec{H} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ H_x & H_y & H_z \end{vmatrix} = \hat{i}\left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right) - \hat{j}\left(\frac{\partial H_z}{\partial x} - \frac{\partial H_x}{\partial z}\right) + \hat{k}\left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right)$$

## ΤΑΧΥΤΗΤΑ ΦΩΤΟΣ

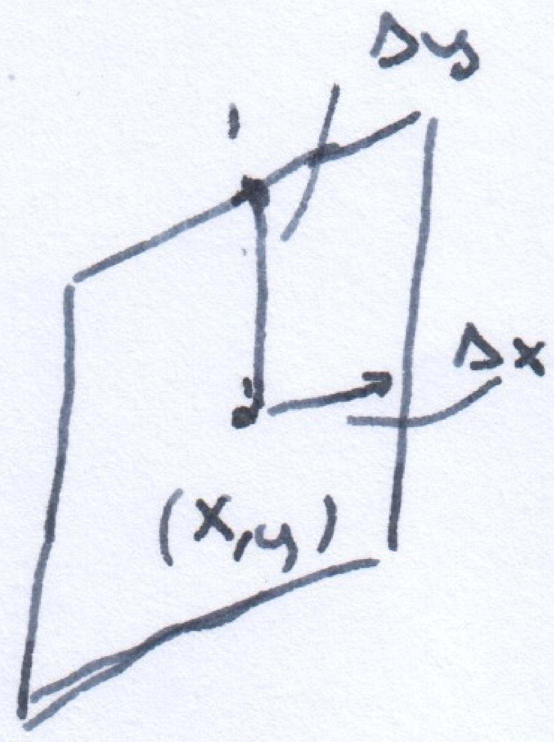
$c^2 = \dfrac{1}{\mu_0\varepsilon_0}$ (στο κενό) $\qquad\qquad$ $\upsilon^2 = \dfrac{1}{\mu\varepsilon}$ (σε μέσο)

# Η/Μ ΚΥΜΑΤΑ

**Επίπεδο κύμα:** Κύμα στο οποίο το μεταβαλλόμενο μέγεθος $\vec{A}$ δεν αλλάζει αν κινηθώ δεξιά (x - άξονας) ή οριζόντια (y-άξονας)

δηλαδή: $$\boxed{\frac{\partial \vec{A}}{\partial x} = 0 \quad κ' \quad \frac{\partial \vec{A}}{\partial y} = 0}$$

A=H




Αν μετακινηθώ κατά Δx δεξιά το κύμα θα είναι πάλι, $H_0 = H(x,y,z)$

Αν μετακινηθώ κατά Δy πάνω πάλι το κύμα θα είναι $H_0 = H(x,y,z)$

**Πρακτικά**

$$\frac{\partial \vec{A}}{\partial x} = 0 \quad κ' \quad \frac{\partial \vec{A}}{\partial y} = 0$$

σημαίνει ότι $\vec{A}(x,y,z,t) = \vec{A}(z,t)$

δηλαδή στη μεταβολή του $\vec{A}$ δεν παίζουν ρόλο οι μεταβλητές (x, y)

**Προσοχή** $\vec{A} = \vec{A}(z,t)$ δεν σημαίνει ότι $A_x$=σταθ, $A_y$=σταθ

**ΕΠΙΠΕΔΟ** $$\boxed{\vec{A}(x,y,z,t) = \hat{A}_x(z,t)\,\hat{u}_x + \hat{A}_y(z,t)\,\hat{u}_y + \hat{A}_z(z,t)\,\hat{u}_z}$$

## Γραμμικά πολωμένο κύμα

$\vec{A} = \vec{A}(z,t)$ ταλαντώνεται μόνο σε έναν άξονα άρα

$$\boxed{\begin{array}{c} \vec{A}_x(z,t) = \text{σταθ} \\ \text{και} \\ \vec{A}_y(z,t) = \text{σταθ} \end{array}}$$

δηλαδή:

$$\boxed{\vec{A} = A_z(z,t)\,\hat{u}_z}$$

# ΕΠΙΠΕΔΑ - ΓΡΑΜΜΙΚΑ ΠΟΛΩΜΕΝΑ Η/Μ ΚΥΜΑΤΑ

Έστω επίπεδο Η/Μ κύμα

$$\vec{H} = H_y(z,t)\,\hat{u}_y \qquad \vec{B} = B_y(z,t)\,\hat{u}_y \quad \text{(πόλωση -y)}$$

$$\vec{D} = D_x(z,t)\,\hat{u}_x \qquad \vec{E} = E_x(z,t)\,\hat{u}_x \quad \text{(πόλωση x)}$$

Δηλαδή:

$$\frac{\partial \vec{H}}{\partial x} = 0 \qquad \frac{\partial \vec{E}}{\partial x} = 0$$

$$\frac{\partial \vec{H}}{\partial y} = 0 \qquad \frac{\partial E}{\partial y} = 0$$

**3ος ΝΟΜΟΣ**

$$\mathrm{curl}\,\vec{E} = -\mu \frac{\partial \vec{H}}{\partial t} \Rightarrow \vec{\nabla} \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t}$$

$$\Rightarrow \begin{vmatrix} \hat{u}_x & \hat{u}_y & \hat{u}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ E_x & E_y & E_z \end{vmatrix} = -\mu \left( \frac{\partial H_x}{\partial t}\hat{u}_x + \frac{\partial H_y}{\partial t}\hat{u}_y + \frac{\partial H_z}{\partial t}\hat{u}_z \right)$$

$$\Leftrightarrow \hat{u}_x \left( \cancelto{0}{\frac{\partial E_z}{\partial y}} - \frac{\partial E_y}{\partial z} \right) + \hat{u}_y \left( \cancelto{0}{\frac{\partial E_z}{\partial x}} - \frac{\partial E_x}{\partial z} \right) + \hat{u}_z \left( \cancelto{0}{\frac{\partial E_z}{\partial x}} - \frac{\partial E_x}{\partial z} \right) =$$

$$\hat{u}_x \left( -\mu \frac{\partial H_x}{\partial t} \right) + \hat{u}_y \left( -\mu \frac{\partial H_y}{\partial t} \right) + \hat{u}_z \left( -\mu \frac{\partial H_z}{\partial t} \right)$$

$$\boxed{\frac{\partial E_y}{\partial z} = \mu \frac{\partial H_x}{\partial t} \qquad \frac{\partial E_x}{\partial z} = -\mu \frac{\partial H_x}{\partial t} \qquad \frac{\partial E_x}{\partial z} = \mu \frac{\partial H_z}{\partial t}}$$

Αλλά:

$$\vec{E} = E_x \hat{u}_x$$

$$\vec{H} = H_y \hat{u}_y$$

άρα

$$\mu \frac{\partial H_x}{\partial t} = \frac{\partial E_y}{\partial y} = 0 \Rightarrow \boxed{\frac{\partial H_x}{\partial t} = 0}$$

$$\mu \frac{\partial H_z}{\partial t} = \frac{\partial E_x}{\partial z} = 0 \Rightarrow \boxed{\frac{\partial H_z}{\partial t} = 0}$$

Άρα

3ος νόμος: $\boxed{\frac{\partial E_x}{\partial z} = -\mu \frac{\partial H_y}{\partial t}}$

όμοια

4ος νόμος: $\boxed{\frac{\partial H_y}{\partial z} = -\epsilon \frac{\partial E_x}{\partial t}}$

Επομένως

$$\frac{\partial E_x}{\partial z} = -\mu \frac{\partial H_x}{\partial t} \Rightarrow \frac{\partial^2 E_z}{\partial z^2} = -\mu \frac{\partial}{\partial z} \left( \frac{\partial}{\partial t} H_y \right)$$

$$\Rightarrow \frac{\partial^2 E_z}{\partial z^2} = -\mu \frac{\partial}{\partial t} \left( \frac{\partial}{\partial z} H_y \right)$$

$$\Rightarrow \frac{\partial^2 E_z}{\partial z^2} = -\mu \frac{\partial}{\partial t} \left( -\epsilon \frac{\partial E_x}{\partial t} \right)$$

$$\Rightarrow \frac{\partial^2 E_z}{\partial z^2} = \mu \epsilon \frac{\partial^2 E_x}{\partial t^2}$$

$$\Rightarrow \boxed{\frac{\partial^2 E_x}{\partial z^2} = \frac{1}{\upsilon^2}\frac{\partial^2 E_x}{\partial t^2} \quad \text{όπου } \frac{1}{\upsilon^2} = \mu\varepsilon \Leftrightarrow \upsilon = \sqrt{\mu\varepsilon}}$$

ΔΗΛΑΔΗ ΟΙ ΕΞΙΣΩΣΕΙΣ MAXWELL ΠΡΟΒΛΕΠΟΥΝ ΔΙΑΔΟΣΗ Η/Μ ΚΥΜΑΤΟΣ

Συνήθως

$$\vec{E}(x,y,z,t) = \underset{\text{(επίπεδο)}}{\vec{E}(z,t)} = \underset{\text{γραμμ. πολωμ.}}{E_x(z,t)}$$

$$= E_0 \sin \frac{2\pi}{\lambda}(\upsilon t - z)$$

κ

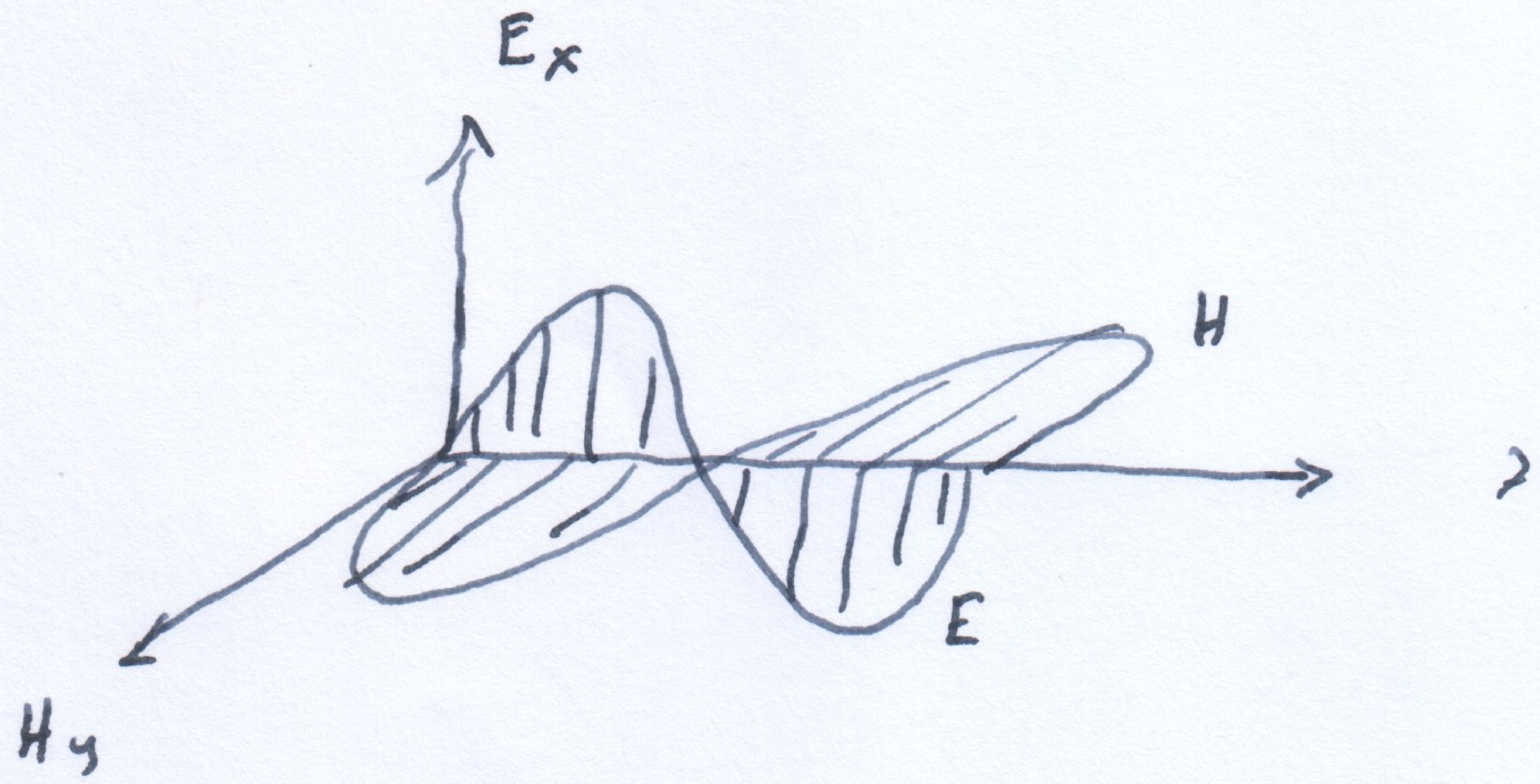
$$\vec{B}(x,y,z,t) = \vec{B}(z,t) = B_y(z,t)$$

$$= B_0 \sin \frac{2\pi}{\lambda}(\upsilon t - z)$$

Δηλαδή:

$$E = E_x = E_0 \sin \frac{2\pi}{\lambda}(\upsilon t - z)$$

$$B = B_y = B_0 \sin \frac{2\pi}{\lambda}(\upsilon t - z)$$




ΚΑΤΕΥΘΥΝΣΗ ΔΙΑΔΟΣΗΣ ΤΩΝ Η/Μ ΚΥΜΑΤΩΝ

$$\vec{P} = \vec{E} \times \vec{H}$$ διάνυσμα Poynting

$$[P] = [\vec{E} \times \vec{H}] = [E] \cdot [H]$$

$$= \frac{V}{m} \cdot \frac{A}{m} = \frac{V \cdot A}{m^2} = \frac{W}{m^2}$$

$$\Rightarrow \boxed{[P] = \frac{W}{m^2}}$$ ισχύς ανά μον. επιφανείας