ΚΙΝΗΜΑΤΙΚΗ ΥΛΙΚΟΥ ΣΗΜΕΙΟΥ

ΘΕΩΡΙΑ

ΚΙΝ 1

ΣΗΜΕΙΟΥΤΑΙ

Ταχύτητα.

$$\vec{v} = \frac{d\vec{r}}{dt}$$

$$\vec{v}_{ave} = \frac{\Delta\vec{r}}{\Delta t}$$

ΚΑΤΑΣΤΑΣΕΙΣ

$$\vec{v} = v_x \hat{u}_x + v_y \hat{u}_y + v_z \hat{u}_z$$

$$v = \sqrt{v_x^2 + v_y^2 + v_z^2}$$

$$v_x = \frac{dx}{dt} \qquad v_y = \frac{dy}{dt} \qquad v_z = \frac{dz}{dt}$$

ΑΥΤΟΝΟΜΕΣ

$$\vec{r} = r\,\hat{u}_r \Rightarrow \frac{d\vec{r}}{dt} = \hat{u}_r \frac{dr}{dt} + r\frac{d\hat{u}_r}{dt}$$

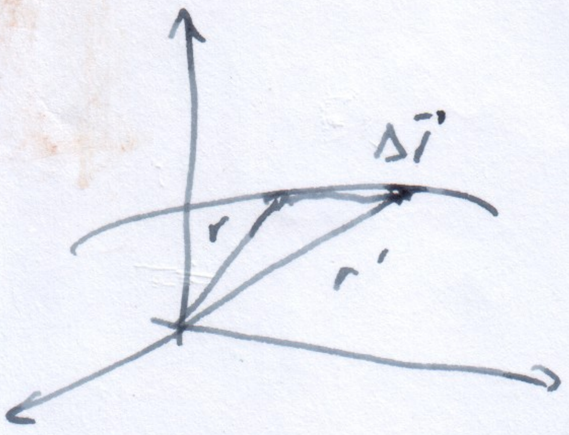
$$\Rightarrow v = \frac{dr}{dt}\hat{u}_r + r\frac{d\hat{u}_r}{dt}$$

ΡΥΘΜΟΣ ΑΛΛΑΓΗΣ / ΘΕΣΗΣ

ΡΥΘΜΟΣ ΑΛΛΑΓΗΣ ΔΙΕΥΘΥΝΣΗΣ

(ΜΟΝΑΔΙΑΙΟΥ ΔΙΑΝΥΣΜΑΤΟΣ)

ΚΙΝΗΜΑΤΙΚΗ ΥΛΙΚΟΥ ΣΗΜΕΙΟΥ



$$\vec{U}_{\mu\varepsilon\sigma} = \frac{\Delta\vec{r}}{\Delta t}$$ (μέση ταχύτητα)

$$\vec{U} = \lim_{\Delta t \to 0} \vec{U}_{\mu\varepsilon\sigma} \Rightarrow \vec{U} = \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$ (στιγμιαία ταχύτητα)

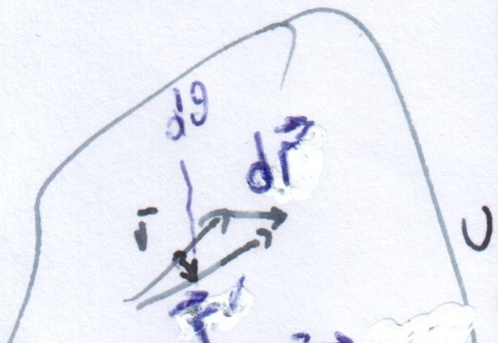
Ανάλυση: $\vec{U} = \hat{U}_T \cdot U$

$U = \lim_{\Delta t \to 0} \frac{\Delta r}{\Delta t} \Rightarrow \vec{U} = \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta s} \frac{\Delta s}{\Delta t} \Rightarrow$

$\Rightarrow \vec{U} = \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta s} \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t}$

Όταν $d\theta \to 0 \rightsquigarrow |\Delta\vec{r}| \sim ds \rightsquigarrow \Delta\vec{r} = \Delta s\, \hat{U}_T$ (διότι $\Delta\vec{r} \uparrow\uparrow \hat{U}_T$)

$\Rightarrow \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta s} = \hat{U}_T$ όπου $\lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = U$



$d\vec{r} \perp \vec{r}$

και

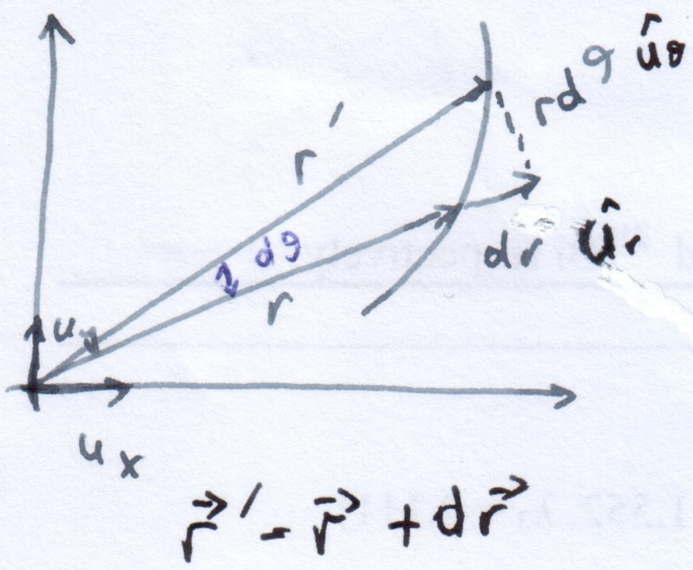
$d\vec{r} // \hat{U}_T$

$\Rightarrow \vec{U} = \hat{U}_T \cdot U$

$\vec{U} = \frac{d\vec{r}}{dt}$

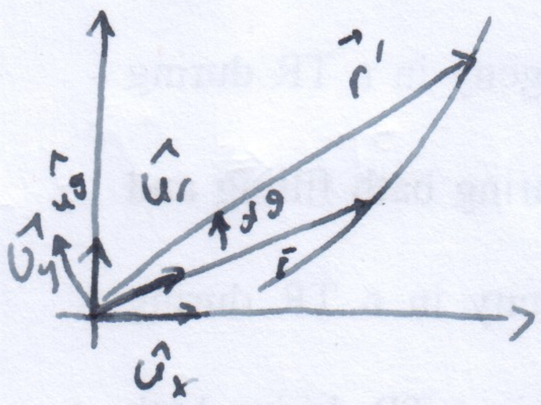
Άρα: Η ΤΑΧΥΤΗΤΑ ΕΙΝΑΙ ΕΦΑΠΤΟΜΕΝΗ ΤΗΣ ΤΡΟΧΙΑΣ

ΤΑΧΥΤΗΤΑ ΣΕ ΑΚΤΙΝΙΚΗ ΣΥΝΤΕΤΑΓΜΕΝΗ (ΚΜ-3)



$\vec{r}' = \vec{r} + d\vec{r}$

$$\vec{v} = \frac{dr}{dt}\hat{u}_r + r\frac{d\hat{u}_r}{dt}$$

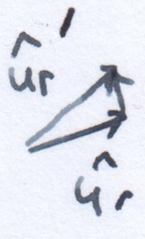


Όταν $d\vartheta \simeq 0 \Rightarrow$ $d\hat{u}_r \perp \hat{u}_r$ Από το ορθογώνιο

$du_r = u_r \cdot \tan d\vartheta \simeq u_r \sin d\vartheta \simeq u_r\, d\vartheta \Rightarrow$ $du_r = u_r d\vartheta$

επειδή όμως $u_r = u\vartheta$ και $d\hat{u}_r \uparrow\uparrow \hat{u}_\theta$ έχουμε ότι

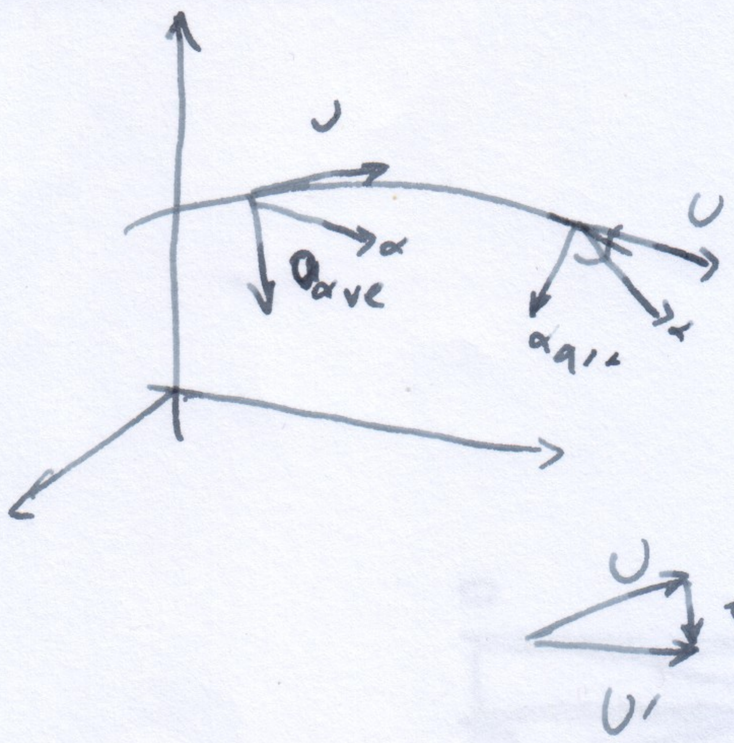
$$d\hat{u}_r = d\vartheta\, \hat{u}_\theta \Rightarrow \frac{d\hat{u}_r}{dt} = \frac{d\vartheta}{dt}\hat{u}_\theta$$



$$\vec{v} = \frac{dr}{dt}\hat{u}_r + r\frac{d\vartheta}{dt}\hat{u}_\theta$$

ΕΠΙΤΑΧΥΝΣΗ

κίνηση



$$\vec{\alpha}_{\mu\varepsilon} = \frac{\Delta\vec{\upsilon}}{\Delta t}$$

$$\Delta\vec{\upsilon} \perp \vec{\upsilon}$$

$$\vec{\alpha}_{\alpha\nu} \perp \vec{\upsilon}$$

$$\vec{\alpha} = \lim_{\Delta t \to 0} \frac{\Delta\vec{\upsilon}}{\Delta t} \Rightarrow \boxed{\vec{\alpha} = \frac{d\vec{\upsilon}}{dt}}$$

$$\boxed{\vec{\upsilon} = \frac{d\vec{r}}{dt}}$$

$$\boxed{\vec{\alpha} = \frac{d^2\vec{r}}{dt^2}}$$

ΕΠΙΤΑΧΥΝΣΗ

$$\vec{\alpha} = \frac{d\vec{v}}{dt}$$

ΚΙΝ Θ

άλλα =...

ΚΑΡΤΕΣΙΑΝΗ

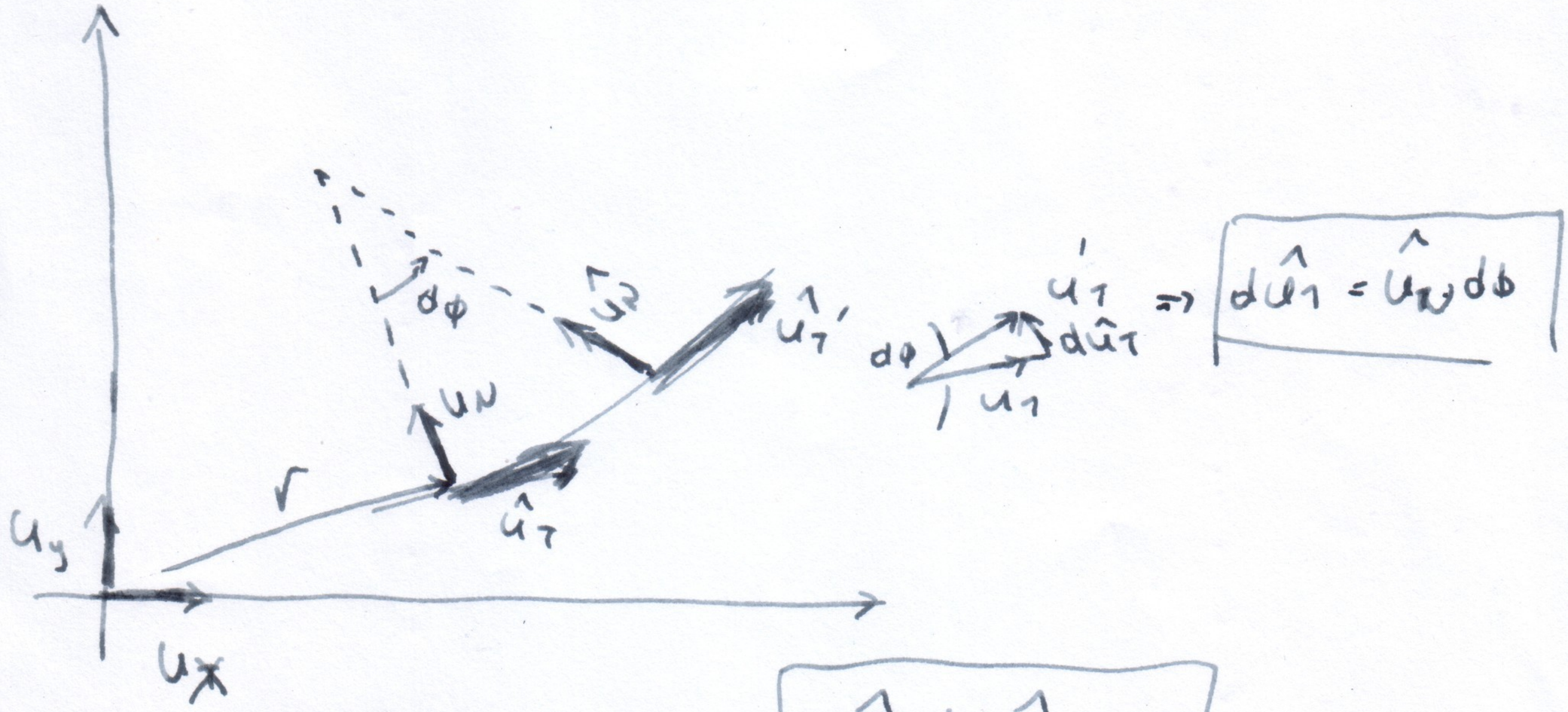
$$\vec{\alpha} = \alpha_x \hat{u}_x + \alpha_y \hat{u}_y + \alpha_z \hat{u}_z$$

$$\alpha = \sqrt{\alpha_x^2 + \alpha_y^2 + \alpha_z^2}$$

$$\alpha_x = \frac{dv_x}{dt} \qquad \alpha_y = \frac{dv_y}{dt} \qquad \alpha_z = \frac{dv_z}{dt}$$

ΕΦΑΠΤΟΜΕΝΙΚΗ - ΑΚΤΙΝΙΚΗ

$$\left.\begin{aligned} \vec{\alpha} &= \frac{d\vec{v}}{dt} \\ \vec{v} &= \hat{u}_\tau \cdot v \end{aligned}\right\} \Rightarrow \boxed{\vec{\alpha} = \frac{dv}{dt}\hat{u}_\tau + v\frac{d\hat{u}_\tau}{dt}}$$



$$d\hat{u}_\tau = \hat{u}_N d\phi$$

$d\phi \to 0 \Rightarrow$

$$d\hat{u}_\tau \perp \hat{u}_\tau$$
$$d\hat{u}_\tau \parallel \hat{u}_N$$

$$d\hat{u}_T = \hat{u}_N \cdot d\varphi$$

$$\alpha = \frac{dU}{dt}\hat{u}_T + U\frac{d\hat{u}_T}{dt} \Rightarrow \boxed{\alpha = \frac{dU}{dt}\hat{u}_T + U\hat{u}_N\frac{d\varphi}{dt}}$$

$$\frac{d\phi}{dt} = \frac{d\phi}{ds}\frac{ds}{dt} = U\frac{d\phi}{ds} \Rightarrow \boxed{\frac{d\phi}{dt} = U\frac{d\phi}{ds}}$$

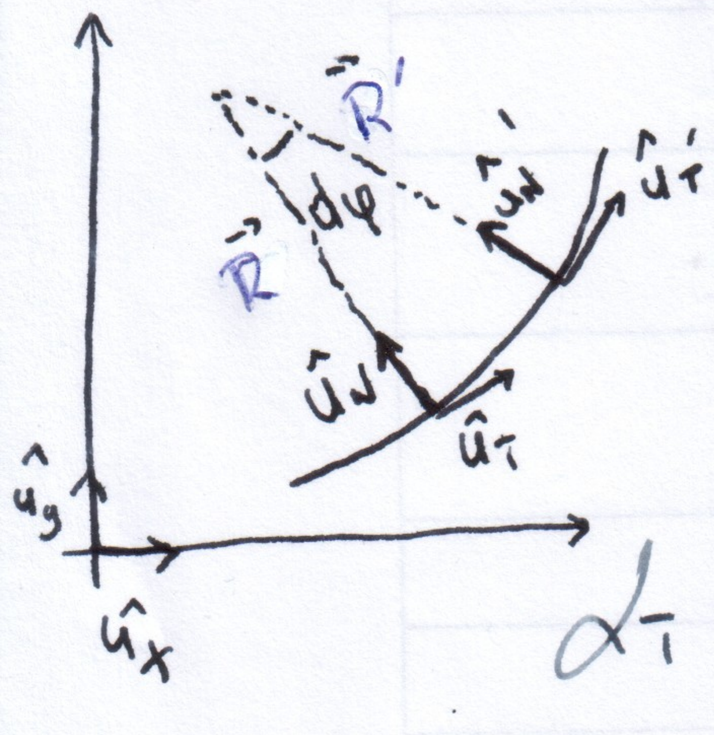
αφού $d\phi = \frac{ds}{R} \Rightarrow \boxed{\frac{d\phi}{ds} = \frac{1}{R}}$

Άρα $\alpha = \frac{dU}{dt}\hat{u}_T + U\cdot U\cdot\frac{1}{R}\cdot\hat{u}_N$

$$\rightarrow \boxed{\alpha = \frac{dU}{dt}\hat{u}_T + \frac{U^2}{R}\cdot\hat{u}_N}$$

επιτάχυνση εφαπτομενική

κεντρομόλος



$\alpha_T = \frac{dU}{dt}\hat{u}_T$ εφαπτομενική

$\alpha_N = \frac{U^2}{R}\hat{u}_N$ κεντρόκεντρος

Επιτάχυνση

$$\vec{\alpha} = \frac{dU}{dt}\hat{U}_T + \frac{U^2}{R}\hat{U}_N$$

Μέτρο επιτάχυνσης

$$\vec{\alpha}_\gamma = \frac{d\vec{\omega}}{dt} \qquad \alpha_\gamma = \frac{d\omega}{dt}$$

- $\alpha_T = \frac{dU}{dt}$, $U = \omega \cdot R$ $\Big\}$ $\alpha_T = \frac{d}{dt}(\omega \cdot R) \Rightarrow \alpha_T = R\frac{d\omega}{dt}$

$$\Rightarrow \boxed{\alpha_T = R \cdot \alpha_\gamma}$$

- $\alpha_N = \frac{U^2}{R}$, $U = \omega \cdot R$ $\Big\}$ $\alpha_N = \frac{\omega^2 \cdot R^{\not 1}}{\not R} \Rightarrow \boxed{\alpha_N = \omega^2 \cdot R}$

- $\vec{\alpha} = \frac{d\vec{U}}{dt}$, $\vec{U} = \vec{\omega} \times \vec{r}$ $\Big\}$ $\vec{\alpha} = \frac{d}{dt}(\vec{\omega} \times \vec{r}) \Rightarrow$

$$\vec{\alpha} = \vec{\omega} \times \frac{d\vec{r}}{dt}$$

$$\Rightarrow \boxed{\vec{\alpha} = \vec{\omega} \times \vec{U}}, \quad \vec{U} = \vec{\omega} \times \vec{r} \quad \Big\} \quad \boxed{\vec{\alpha} = \vec{\omega} \times (\vec{\omega} \times \vec{r})}$$

σημ.

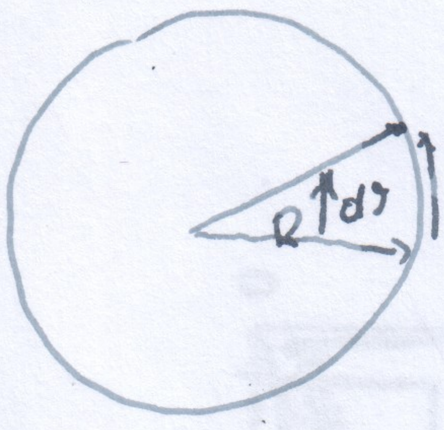
$\vec{\alpha} = \vec{\omega} \times \vec{U}$

$\vec{\omega} \times \vec{U} \perp \vec{U}$ $\Big|$ $\vec{\alpha} \perp \vec{U}$ / $\vec{\alpha} \perp \vec{\omega}$ } πρώτα κίνημα

Κυκλική κίνηση

ΚΜΟ

$$|\vec{r}| = |\vec{R}| = \text{σταθ.}$$



α) $\vec{U} = \frac{d\vec{r}}{dt}\hat{u}_r + r\frac{d\vartheta}{dt}\hat{u}_\vartheta \xrightarrow{r=\sigma\tau\alpha\theta} U = r\frac{d\vartheta}{dt}\hat{u}_\vartheta$

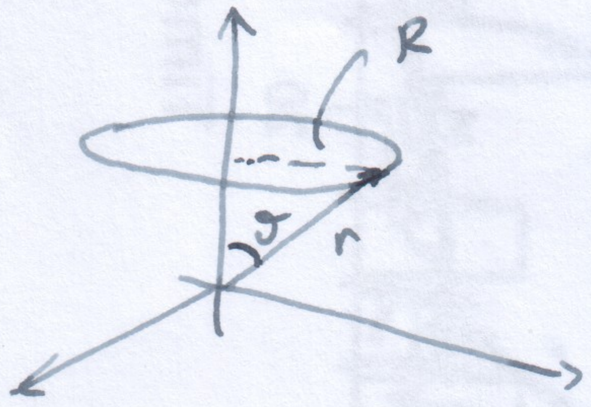
$$\Rightarrow \boxed{\vec{U} = \omega \cdot r \cdot u_T}$$

β) $U = \frac{ds}{dt}$

$ds = R \cdot d\vartheta$

$$\left\{ U = \frac{R\,d\vartheta}{dt} \rightarrow \boxed{U = R\frac{d\vartheta}{dt}} \right.$$

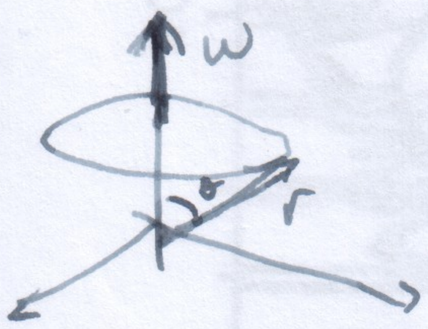
Γωνιακή ταχύτητα $\boxed{\omega = \frac{d\vartheta}{dt}}$



$R = r\sin\vartheta$

$U = R\frac{d\vartheta}{dt}$

$$\left\{ U = r\sin\vartheta\frac{d\vartheta}{dt} \right.$$



$$\Rightarrow \boxed{\vec{U} = \vec{\omega} \times \vec{r}}$$

$$\boxed{U = \omega \cdot r\sin\vartheta}$$

Αυτή η σχέση ισχύει μόνο για κυκλική κίνηση $\Rightarrow R = ct$ $r = ct$

Συχνότητα $f = \frac{\Delta N}{\Delta t}$ $\boxed{f = \frac{1}{T}}$

$$\boxed{U = \omega \cdot R \qquad U = \frac{2\pi R}{T} \qquad U = 2\pi R f}$$