3.6 $m = 2kg$

$F = 24t + 4$ (SI)

$t=0$ $\upsilon = 6 m/s$, $x_0 = 1m$

$\upsilon = \upsilon(t)$ $x = x(t)$

$$F = m\frac{d\upsilon}{dt} \Rightarrow 2\frac{d\upsilon}{dt} = 24t + 4 \Rightarrow \frac{d\upsilon}{dt} = 12t + 2 \Rightarrow$$

$$d\upsilon = 12t\,dt + 2dt \Rightarrow \int_6^\upsilon d\upsilon = 12\int_0^t t\,dt + 2\int_0^t dt$$

$$\Rightarrow \upsilon - 6 = 12\left[\frac{t^2}{2}\right]_0^t + 2[t]_0^t \Rightarrow \boxed{\upsilon = 6 + 6t^2 + 2t}$$

$$\upsilon = \frac{dx}{dt} \Rightarrow dx = \upsilon\,dt \Rightarrow dx = (6 + 6t^2 + 2t)\,dt$$

$$\Rightarrow dx = 6dt + 6t^2dt + 2t\,dt \Rightarrow \int_1^x dx = 6\int_0^t dt + 6\int_0^t t^2dt + \int_0^t 2t\,dt$$

$$\Rightarrow x - 1 = 6[t]_0^t + \cancel{6}^{\,2}\left[\frac{t^3}{\cancel{3}}\right]_0^t + 2\left[\frac{t^2}{2}\right]$$

$$\Rightarrow \boxed{x = 6t + 2t^3 + t^2 + 1}$$

3.8

$F = \lambda \frac{1}{v}$ $\lambda = \text{const}$ $v = v(t)$

$t = 0\,s$ $v = v_0$ $\alpha = \alpha(t)$

$F \uparrow\uparrow v$

$$m \frac{dv}{dt} = \lambda \frac{1}{v} \Rightarrow m\,dv \cdot v = \lambda\,dt \rightarrow m \int_{v_0}^{v} dv\, v = \lambda \int_0^t dt$$

$$\rightarrow m \left[\frac{v^2}{2}\right]_{v_0}^{v} = \lambda \left[t\right]_0^t \rightarrow m \frac{v^2 - v_0^2}{2} = \lambda t$$

$$\Rightarrow m v^2 - m v_0^2 = 2\lambda t \rightarrow m v^2 = 2\lambda t + m v_0^2$$

$$\Rightarrow \boxed{v = \sqrt{\frac{2\lambda t}{m} + v_0^2}}$$

$F = m\alpha \Rightarrow$ $F = \lambda \frac{1}{v} \Rightarrow$

$$\Rightarrow F = \lambda \left(\sqrt{\frac{2\lambda t}{m} + v_0^2}\right)^{-1}$$

3.8

$F = -kv^2$, $k = \text{const}$

$t = 0s \Rightarrow v = v_0$

$\vec{F} \uparrow\downarrow \vec{v}$

$v = v(t) \qquad \alpha = \alpha(t)$

$$F = m\frac{dv}{dt} \Rightarrow m\frac{dv}{dt} = -kv^2 \Rightarrow \frac{dv}{v^2} = -\frac{k}{m}\,dt$$

$$\Rightarrow \int_{v_0}^{v} \frac{dv}{v^2} = -\frac{k}{m}\int_0^t dt \Rightarrow -\left[v^{-1}\right]_{v_0}^{v} = -\frac{k}{m}\left[t\right]_0^t$$

$$\Rightarrow v^{-1} - v_0^{-1} = \frac{k}{m}t \Rightarrow v^{-1} = v_0^{-1} + \frac{k}{m}t$$

$$\Rightarrow \boxed{v = \frac{1}{v_0^{-1} + \frac{k}{m}t}}$$

$$\alpha = \frac{F}{m} \Rightarrow \alpha = -\frac{kv^2}{m} \Rightarrow \boxed{\alpha = -\frac{k}{m}\left(\frac{1}{v_0^{-1} + \frac{k}{m}t}\right)^2}$$

(310) κίνηση σε ρευστό   τέλος $υ=0\,m/s$ (4)

$$\Sigma F = F - k_η υ$$

$υ = υ(t)$ $t_{1/2}$ Βαρύτητα αμελείται

$$m\frac{dυ}{dt} = F - k_η υ \Rightarrow \frac{dυ}{F-k_η υ} = +\frac{1}{m}dt \Rightarrow \boxed{\frac{dυ}{k_η υ - F} = \frac{1}{m}dt}$$

$$\boxed{ξ = k_η υ - F} \Rightarrow dξ = k_η dυ \Rightarrow \boxed{dυ = \frac{dξ}{k_η}}$$

Άρα $\frac{dξ}{ξ} = -\frac{1}{m}dt \Rightarrow \frac{1}{k_η}\int_{-F}^{-F+k_η υ}\frac{dξ}{ξ} = -\frac{k_η}{m}\int_0^t dt$

$$\Rightarrow \ln(-F+k_η υ) - \ln(-F) = -\frac{k_η}{m}t$$

$$\Rightarrow \ln\frac{-F+k_η υ}{-F} = -\frac{1}{m}t \Rightarrow \frac{-F+k_η υ}{-F} = e^{-\frac{k_η}{m}t}$$

$$\Rightarrow -F + k_η υ = -F e^{-k_η t/m} \Rightarrow k_η υ = F - F e^{-k_η t/m}$$

$$\Rightarrow k_η υ = F\left(1 - e^{-k_η t/m}\right) \Rightarrow \boxed{υ = \frac{F}{k_η}\left(1 - e^{-k_η t/m}\right)}$$

$υ_0 = \frac{F}{k_η}$

$$\frac{υ_0}{2} = υ_{t_{1/2}} = \frac{F}{k_η}F\left(1 - e^{-k_η t_{1/2}/m}\right) \Rightarrow \frac{υ_0}{2} = υ_0\left(1 - e^{-k_η t_{1/2}/m}\right)$$

$$\Rightarrow \frac{1}{2} = \left(1 - e^{-k_η t_{1/2}/m}\right) \Rightarrow \ln 1 - \ln 2 = e^{-k_η t_{1/2}/m}$$

$$-\ln 2 = -\frac{k_1 t_{1/2}}{m} \Rightarrow k_1 t_{1/2} = m\ln 2 \Rightarrow \boxed{t_{1/2} = \frac{m}{k_1}\ln 2}$$

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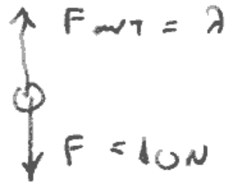
$m = 2kg$

$t = 0s \quad \upsilon = 0 m/s$



$, \lambda = 2 \frac{N \cdot s}{m}$

(α) $\alpha(t=0) = ;$

(β) $\upsilon_{ορ} = ;$

(γ) $\alpha(t=1s) \quad \upsilon(t=1s)$

(δ) Διάγραμμα $\upsilon = \upsilon(t)$

$$\Sigma F = m\alpha \Rightarrow F - \lambda\upsilon = m\alpha \Rightarrow \boxed{\alpha = \frac{F}{m} - \frac{\lambda}{m}\upsilon}$$

(α) $t=0 \quad \upsilon = 0 \Rightarrow \boxed{\alpha(t=0) = \frac{F}{m}} \boxed{= 5 m/s^2}$

(β) $\alpha = 0$, $\upsilon = \upsilon_{ορ}$: $\alpha = \frac{F}{m} - \frac{\lambda}{m}\upsilon \Rightarrow 0 = \frac{F}{m} - \frac{\lambda}{m}\upsilon_{ορ}$

$$\Rightarrow \frac{F}{\cancel{m}} = \frac{\lambda}{\cancel{m}}\upsilon_{ορ} \Rightarrow \boxed{\upsilon_{ορ} = \frac{F}{\lambda}} \boxed{= 5 m/s}$$

(γ) $\Sigma F = m\frac{d\upsilon}{dt} \Rightarrow \frac{F}{m} - \frac{\lambda}{m}\upsilon = \cancel{m}\frac{d\upsilon}{dt} \Leftrightarrow \alpha = \frac{F}{m} - \frac{\lambda}{m}\upsilon$

$$\Leftrightarrow \frac{d\upsilon}{dt} = \frac{F}{m} - \frac{\lambda}{m}\upsilon \Rightarrow \frac{d\upsilon}{\frac{F}{m} - \frac{\lambda}{m}\upsilon} = dt \Rightarrow$$

$$\Rightarrow \boxed{\frac{d\upsilon}{\frac{\lambda}{m}\upsilon - \frac{F}{m}} = -dt}$$

$\xi = \frac{\lambda}{m}\upsilon - \frac{F}{m} \quad d\xi = \frac{\lambda}{m}d\upsilon$

$$\boxed{d\upsilon = \frac{m}{\lambda}d\xi}$$

$$\frac{du}{\frac{\lambda}{m}u-\frac{F}{m}} = -dt \Rightarrow \frac{\frac{m}{\lambda}d\xi}{\xi} = -dt \Rightarrow \frac{d\xi}{\xi} = -\frac{\lambda}{m}dt$$

$$\Rightarrow \int_{-\frac{F}{m}}^{\frac{\lambda}{m}u-\frac{F}{m}} \frac{d\xi}{\xi} = -\frac{\lambda}{m}\int_0^t dt \rightarrow \ln\left(\frac{\lambda}{m}u-\frac{F}{m}\right)-\ln\left(-\frac{F}{m}\right) = -\frac{\lambda}{m}t$$

$$\Rightarrow \ln\left(\frac{\frac{\lambda}{m}u-\frac{F}{m}}{-\frac{F}{m}}\right) = -\frac{\lambda}{m}t \Rightarrow$$

$$\Rightarrow \frac{\frac{\lambda}{m}u-\frac{F}{m}}{-\frac{F}{m}} = e^{-\lambda/m\, t} \Leftrightarrow \frac{\lambda}{m}u-\frac{F}{m} = -\frac{F}{m}e^{-\lambda/m\, t}$$

$$\Rightarrow \frac{\lambda}{m}u = \frac{F}{m}-\frac{F}{m}e^{-\lambda/m\, t} \Rightarrow \frac{\lambda}{\cancel{m}}u = \frac{F}{\cancel{m}}\left(1-e^{-\frac{\lambda}{m}t}\right)$$

$$\Rightarrow \boxed{u = \frac{F}{\lambda}\left(1-e^{-\frac{\lambda t}{m}}\right)} \boxed{=5\left(1-e^{-t}\right)}$$

$t=1s \qquad u = 5\left(1-e^{-1}\right) = 3,15\ m/s$

$$\alpha = \frac{F}{m}-\frac{\lambda}{m}u \Rightarrow \alpha = 5-3,15 \Rightarrow$$

$$\Rightarrow \alpha = 1,85\ m/s^2$$

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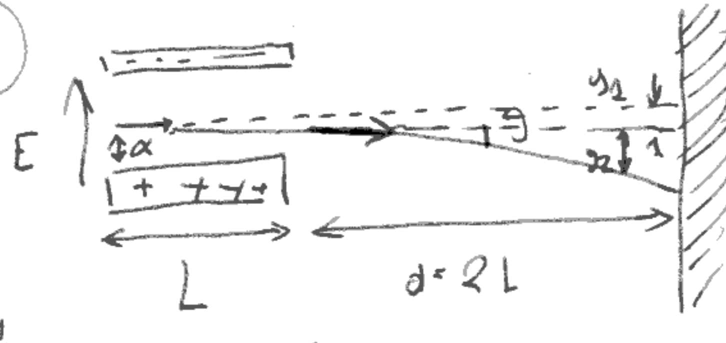
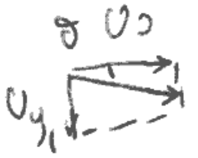
(7)





(α) Υπολογίστε το y_1, y_2

(β) Την ελάχιστη v_0 ώστε να εξέρχονται του πεδίου

(γ) ατελ λπ.

x: ΕΟΚ $\Sigma F_x = 0 \Rightarrow \boxed{\begin{array}{l} v_x = v_0 \\ x = v_0 t \end{array}}$

$x = L,\ t = t_1 \Rightarrow L = v_0 \cdot t_1 \Rightarrow \boxed{t_1 = \frac{L}{v_0}}$

y: ΕΟΕ $\Sigma F_y = E\cdot q \Rightarrow m\alpha_y = Eq \Rightarrow \boxed{\alpha_y = \frac{E\cdot q}{m}}$

$$\boxed{\begin{array}{l} v_y = \alpha_y t \\ y = \frac{1}{2}\alpha_y t^2 \end{array}}$$

ΟΤΑΝ $t = t_1$, $y = y_1$. Άρα

$$y_1 = \frac{1}{2}\alpha_y t_1^2 \Rightarrow \boxed{y_1 = \frac{Eq}{2m}\frac{L^2}{v_0^2}}$$

Έξω από το πεδίο κάνει ΕΟΚ μέχρι το πέτασμα.

$$\tan\theta = \frac{v_{y_1}}{v_0} \Rightarrow \tan\theta = \frac{\alpha_y t_1}{v_0} \Rightarrow \tan\theta = \frac{\frac{Eq}{m}\frac{L}{v_0}}{v_0}$$

$$\Rightarrow \boxed{\tan\theta = \frac{EqL}{m v_0^2}}$$

Αλλά

$$\boxed{\tan\theta = \frac{y_2}{2L}}$$

Άρα $\frac{y_2}{2L} = \frac{EqL}{mv_0^2} \Rightarrow \boxed{y_2 = \frac{2EqL^2}{mv_0^2}}$

Οριακά για να εξέρχεται του πεδίου θα πρέπει

$$y_{1_{cr}} = \alpha \Rightarrow \frac{Eq}{2m}\frac{L^2}{v_{0_{cr}}^2} = \alpha \Rightarrow v_{0_{cr}}^2 = \frac{EqL^2}{2m\alpha}$$

$$\Rightarrow v_{0_{cr}} = \sqrt{\frac{EqL^2}{2m\alpha}} \Rightarrow \boxed{v_{0cr} = L\sqrt{\frac{Eq}{2m\alpha}}}$$

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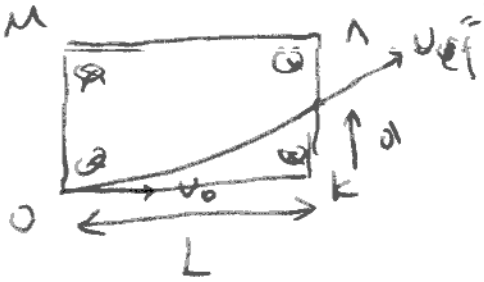
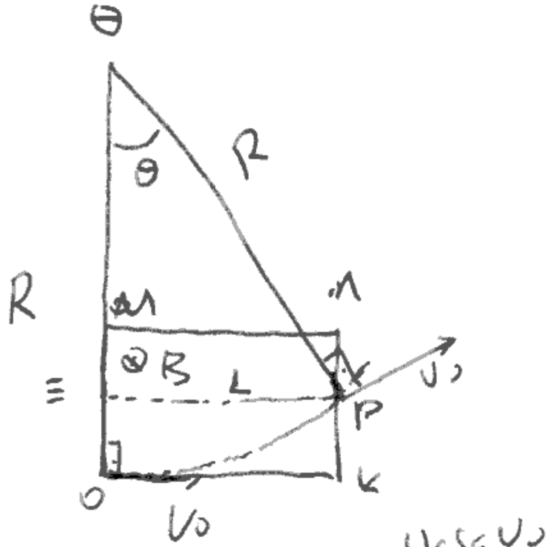


$=v_0$ σωματίδια α (m, q γνωστά), B εισέρχονται στο O με $v=v_0$ και διέρχεται ΟΚ.

(α) d = ;

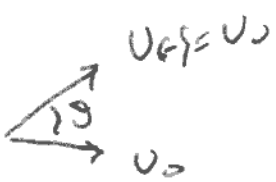
(β) γωνία $v_{εξ}$ με v_0



$$R = \frac{mv_0}{B \cdot q}$$

<u>Τριγωνομετρ:</u> $\sin\theta = \frac{L}{R}$, $R = \frac{mv_0}{Bq}$ $\Rightarrow$

γωνία με πλευρά κλίσεις



$$\Rightarrow \boxed{\sin\theta = \frac{LBq}{mv_0}}$$

$d = KP$

$KP = \Theta O - \Theta E$

$d = \Theta O - \Theta E$

$\Theta O = R$

$\Theta E = R\cos\theta$

$d = R - R\cos\theta \Rightarrow$

$$\boxed{d = R(1-\cos\theta)}$$

$\cos^2\theta + \sin^2\theta = 1 \Rightarrow \cos^2\theta = 1 - \sin^2\theta$

$\sin\theta = \frac{LB q}{m v_0}$

$\cos^2\theta = 1 - \frac{L^2 B^2 q^2}{m^2 v_0^2}$

$$\Rightarrow \boxed{\cos\theta = \sqrt{1 - \frac{L^2 B^2 q^2}{m^2 v_0^2}}}$$

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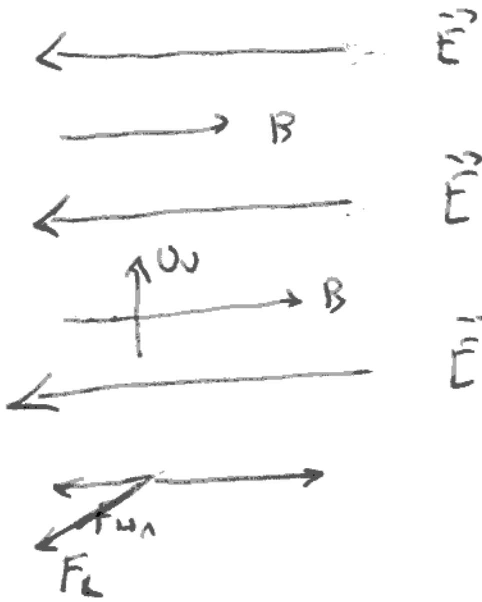

$\vec{E} \parallel \vec{B}$

Σωματίδιο m, q εκτοξεύεται

με $v_0 \perp \vec{E}, \vec{B}$

(α) Να βρεθεί το βήμα της έλικας

(β) Να βρεθεί το $b_v - b_{(v-1)}$

$F_{ηλ} = E \cdot q \Rightarrow m a_x = Eq \Rightarrow \boxed{a_x = \frac{Eq}{m}}$ $\boxed{x = \frac{1}{2} a_x t^2}$

$F_L = qvB \Rightarrow \boxed{R = \frac{mv}{Bq}}$ $\boxed{T = \frac{2\pi m}{Bq}}$ $\boxed{v_x = a_x t}$

$$\left.\begin{aligned} U &= \sqrt{U_x^2 + U_y^2} \\ U_y &= U_0 \\ U_x &= \alpha_x t \end{aligned}\right\} \quad U = \sqrt{U_0^2 + \alpha_x^2 t^2} \Rightarrow \boxed{U = \sqrt{U_0^2 + \frac{E^2 q^2}{m^2} t^2}}$$

$$R = \frac{mU}{Bq} \Rightarrow R = \frac{m}{Bq}\sqrt{U_0^2 + \frac{E^2 q^2}{m^2} t^2}$$

$$\Rightarrow R = \sqrt{\frac{m^2 U_0^2}{B^2 q^2} + \frac{E^2 q^2}{B^2 q^2}\frac{\cancel{m^2}}{\cancel{m^2}} t^2}$$

$$\Rightarrow \boxed{R = \sqrt{\frac{p^2}{B^2 q^2} + \frac{E^2 q^2}{B^2 q^2} t^2}}$$

<u>βήμα έλικας</u>

$$\boxed{\delta\beta = \beta_\nu - \beta_{(\nu-1)}}$$

$$\beta_\nu = \nu^2 \frac{1}{2}\alpha_x T^2 \quad (t = \nu T)$$

$$\beta_{(\nu-1)} = (\nu-1)^2 \frac{1}{2}\alpha_x T^2 \quad (t = (\nu-1)T)$$

Άρα: $\delta\beta = \frac{1}{2}\alpha_x\left[(\nu T)^2 - ((\nu-1)T)^2\right] = \frac{1}{2}\alpha_x\left[\nu^2 T^2 - (\nu-1)^2 T^2\right]$

$$\Rightarrow \boxed{\delta\beta = \frac{1}{2}\alpha_x T^2\left[\nu^2 - (\nu-1)^2\right]}$$