$F_{αντ} = -bv$

ΦΘΙΝΟΥΣΑ ΤΑΛΑΝΤΩΣΗ

$$m\ddot{x} = -Dx - bv \Rightarrow m\ddot{x} = -Dx - b\dot{x} \Rightarrow m\ddot{x} + Dx + b\dot{x} = 0$$

$$\Rightarrow m\ddot{x} + b\dot{x} + Dx = 0 \Rightarrow$$

$$\rightarrow \boxed{\ddot{x} + \frac{b}{m}\dot{x} + \frac{D}{m}x = 0} \quad (1)$$

Επιλέγω λύση $x = Ae^{kt}$ $\quad [A] = m$

$$(1): \quad Ak^2e^{kt} + \frac{b}{m} \cdot ke^{kt} + \frac{D}{m}Ae^{kt} = 0$$

$$\Rightarrow Ae^{kt}\left(k^2 + \frac{b}{m}k + \frac{D}{m}\right) = 0$$

$$\Rightarrow \boxed{k^2 + \frac{b}{m}k + \frac{D}{m} = 0}$$

Διακρίνουσα:

$$\Delta = \frac{b^2}{m^2} - 4\frac{D}{m} \Rightarrow \Delta = \frac{b^2 - 4mD}{m^2}$$

$$k = \frac{-\frac{b}{m} \pm \sqrt{\Delta}}{2} = \frac{-\frac{b}{m} \pm \sqrt{\frac{b^2-4mD}{m^2}}}{2}$$

$$\Rightarrow k = -\frac{b}{2m} \pm \frac{1}{2}\sqrt{\frac{b^2-4mD}{m^2}}$$

$$\Rightarrow k = -\frac{b}{2m} \pm \sqrt{\frac{b^2-4mD}{4m^2}}$$

$$\Rightarrow k = -\frac{b}{2m} \pm \sqrt{\frac{b^2}{4m^2} - \frac{4mD}{4m^2}}$$

$$\Rightarrow \boxed{k = -\frac{b}{2m} \pm \sqrt{\left(\frac{b}{2m}\right)^2 - \frac{D}{m}}}$$

Περίπτωση 3 Απλή Αρμονική Ταλάντωση με απόσβεση.

$\left(\frac{b}{2m}\right)^2 < \frac{D}{m}$

$$k = -\frac{b}{2m} \pm \sqrt{(-1)\left(\frac{D}{m} - \left(\frac{b^2}{2m}\right)\right)}$$

$$\Rightarrow \boxed{k = \frac{-b}{2m} \pm j\sqrt{\frac{D}{m} - \left(\frac{b}{2m}\right)^2}}$$

Λύση της Δ.Ε.Τ. είναι:

$$x = Ae^{kt} \Rightarrow x = Ae^{\frac{-b}{2m}t} e^{\pm j\sqrt{\frac{D}{m} - \left(\frac{b}{2m}\right)^2}\, t}$$

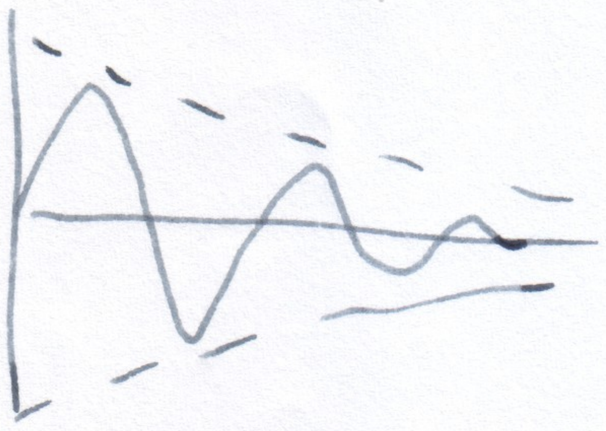
$$\Rightarrow x = Ae^{\frac{-b}{2m}t}\left(e^{j\omega' t} + e^{-j\omega' t}\right), \quad \omega' = \sqrt{\frac{D}{m} - \left(\frac{b}{2m}\right)^2}$$

$$\Rightarrow \boxed{\begin{array}{l} x = Ae^{-\Lambda t}\left(e^{j\omega' t} + e^{j\omega' t}\right) \\ \\ \Lambda = -\frac{b}{2m} \qquad \omega' = \sqrt{\frac{D}{m} + \left(\frac{b}{2m}\right)^2} \end{array}}$$

ή

$$\boxed{\begin{array}{l} x = e^{-\Lambda t}\left(A_1 e^{j\omega' t} + A_2 e^{j\omega' t}\right) \\ \\ \Lambda = -\frac{b}{2m} \qquad \omega' = \sqrt{\frac{D}{m} - \left(\frac{b}{2m}\right)^2} \end{array}}$$

$A_1 = \frac{A}{2_1} e^{i\Phi}$ $\qquad$ $A_2 = -\frac{A}{2_1} e^{-i\Phi}$



Περίπτωση 2: Κρίσιμη Απόσβεση

$\left(\frac{b}{2m}\right)^2 = \frac{D}{m}$: $k = -\frac{b}{2m}$ (διπλή ρίζα)

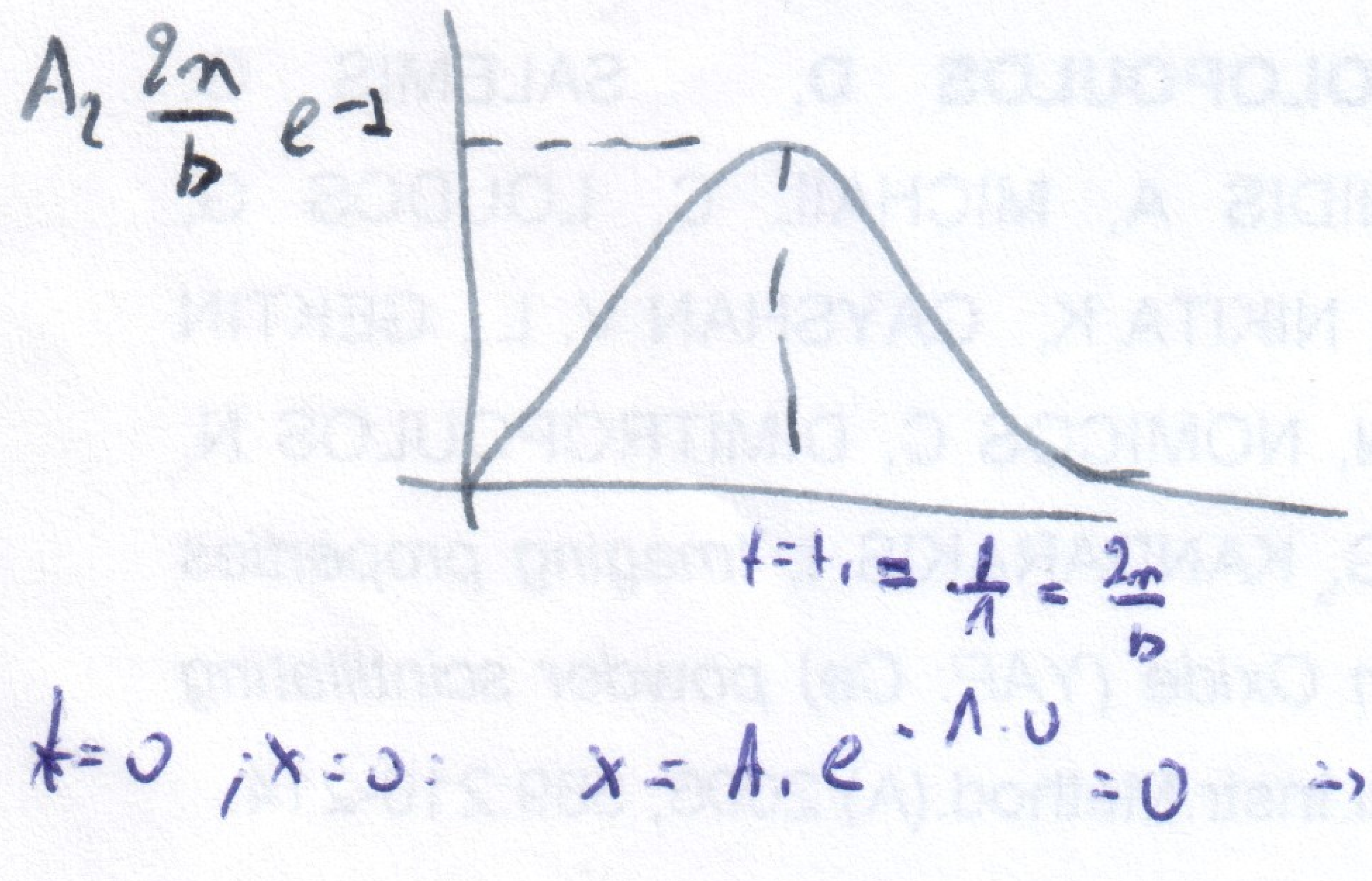
Εφαρμόζω λύση της δ.ε.

$A = (A_1 + A_2 t)$ $x = (A_1 + A_2 t)e^{kt} \Rightarrow x = (A_1 + A_2 t)e^{-(b/2m)t}$

$\Lambda = -\frac{b}{2m}$ $x = (A_1 + A_2 t)\, e^{-\Lambda t}$

Το σύστημα επιστρέφει στο σημείο μηδενικής απόκλισης στον ελάχιστο χρόνο (μετά από ταλάντωση)

Λύση για $t=0$ $x=0$ $x = A_2 t e^{-\Lambda t}$



Για $t=0$, $x=0$: $x = A_1 e^{-\Lambda \cdot 0} = 0 \Rightarrow A_1 = 0 \Rightarrow$

$\Rightarrow x = A_2 t e^{-\Lambda t}$ | $\Lambda = \frac{b}{2m}$

x_{max}: $t = t_1$ $x = x_{max} \Rightarrow \dot{x} = 0$

$\dot{x} = 0 \Rightarrow A_2 \cdot t_1 (-\Lambda) e^{-\Lambda t_1} + A_2 e^{-\Lambda t_1} = 0 \Rightarrow A_2 e^{-\Lambda t_1}\left(t_1(-\Lambda) + 1\right) = 0$

$\Rightarrow 1 - \Lambda t_1 = 0 \Rightarrow t_1 = \frac{1}{\Lambda} \Rightarrow t_1 = \frac{2m}{b}$

$t = t_1$: $x_{max} = A_2 \cdot t_1 e^{-\Lambda t_1} \Rightarrow$ $\boxed{X_{max} = A_2 \frac{2m}{b} e^{-1}}$

Περίπτωση 3: Υπεραπόσβεση

$\left(\frac{b}{2m}\right)^2 > \left(\frac{D}{m}\right)$: $\boxed{k = -\frac{b}{2m} \pm \sqrt{\left(\frac{b}{2m}\right)^2 - \frac{D}{m}}}$

Λύση της δ.ε.τ.

$$X = Ae^{kt} \Rightarrow x = A e^{-\left(\frac{b}{8m}\right)t} \left(e^{\sqrt{\left(\frac{b}{2m}\right)^2 - \frac{D}{m}}\, t} + e^{-\sqrt{\left(\frac{b}{2m}\right)^2 - \frac{D}{m}}\, t}\right)$$

$$\Rightarrow X = A e^{-\Lambda t}\left(e^{\omega' t} + e^{-\omega' t}\right), \quad \omega' = \sqrt{\left(\frac{b}{2m}\right)^2 - \frac{D}{m}}$$

Εισάγω δύο παραμέτρους A_1 και A_2 αντί της A

$$X = e^{-\Lambda t}\left(A_1 e^{\omega' t} + A_2 e^{-\omega' t}\right)$$

και τις

$$B = A_1 + A_2, \quad C = A_1 - A_2$$

$$X = e^{-\Lambda t}\left(\frac{B}{2}\left(e^{\omega' t} + e^{-\omega' t}\right) + \frac{C}{2}\left(e^{\omega' t} - e^{-\omega' t}\right)\right)$$

$$\Rightarrow \boxed{X = e^{-\Lambda t}\left(B \cosh \omega' t + C \sinh \omega' t\right)}$$

Λύση $x=0$, $t=0$ $\quad x = C e^{-\lambda t} \sinh k_1 t$

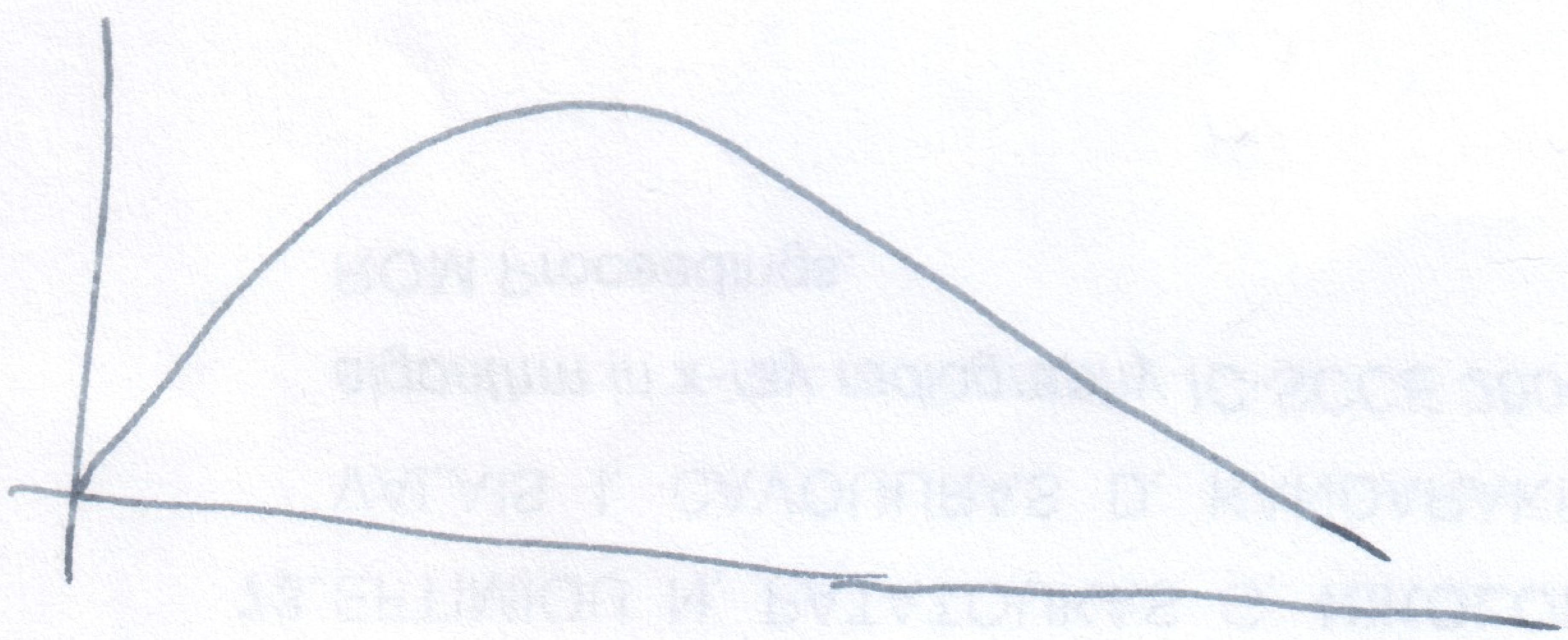
$t=0 \quad x=0 \Rightarrow \quad x = e^{-\lambda t}\left(0 + C \sinh k_1 t\right)$

$$\rightarrow \boxed{x = C e^{-\lambda t} \sinh k_1 t}$$

$$C = A_1 - A_2$$

$$k_1 = \sqrt{\left(\frac{b}{2m}\right)^2 - \frac{D}{m}}$$

$$\lambda = -\frac{b}{2m}$$



Το σύστημα τελικά δεν
[illegible] επιστρέφει
παρά πολύ αργά στη ΘΙ