

6 July 2015

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i)

```
<< VectorAnalysis`;  
SetCoordinates[Cartesian[x, y, z]];  
ClearAll[x, y, z, a];  
f[x_, y_] := Exp[-x^2 - y^2]  
a = {-1, 1, 0}; a0 = a / Sqrt[a.a];  
Print["grad f = ", Grad[f[x, y]]]  
gradfP = Grad[f[x, y]] /. {x -> 1, y -> 1};  
Print["Unit vector = ", a0]  
Print["grad f at point P : ", gradfP]  
Print[  
    "directional derivative at point P : ", gradfP.a0]
```

$$\text{grad } f = \{-2 e^{-x^2-y^2} x, -2 e^{-x^2-y^2} y, 0\}$$

$$\text{Unit vector} = \left\{-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right\}$$

$$\text{grad } f \text{ at point } P : \left\{-\frac{2}{e^2}, -\frac{2}{e^2}, 0\right\}$$

directional derivative at point P : 0

ii)

```

ClearAll[x, y, z, t];
x[t_] := 2 Cos[t]; y[t_] := 2 Sin[t]
Print["x(t) = ", x[t]]; Print["y(t) = ", y[t]]
xd[t_] := D[x[t], t]; yd[t_] := D[y[t], t]
Print["x'(t) = ", xd[t]]; Print["y'(t) = ", yd[t]]
P[t_] := -x[t]; Q[t_] := y[t]
Print["P(t) = ", Simplify[P[t]]]
Print["Q(t) = ", Simplify[Q[t]]]
Print["P(t)x'(t)+Q(t)y'(t) = ",
      Simplify[P[t] xd[t] + Q[t] yd[t]]]
w = Integrate[P[t] xd[t] + Q[t] yd[t], {t, 0, Pi / 2}];
Print["Linear Integral: ", w]

```

$$x(t) = 2 \cos[t]$$

$$y(t) = 2 \sin[t]$$

$$x'(t) = -2 \sin[t]$$

$$y'(t) = 2 \cos[t]$$

$$P(t) = -2 \cos[t]$$

$$Q(t) = 2 \sin[t]$$

$$P(t)x'(t)+Q(t)y'(t) = 8 \cos[t] \sin[t]$$

$$\text{Linear Integral: } 4$$

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i)

```
Solve[x^2 == 2 - x^2, x]
```

```
Integrate[x - y, {y, x^2, 2 - x^2}]
```

```
Print["Double Integral: ",
```

```
  Integrate[x - y, {x, -1, 1}, {y, x^2, 2 - x^2}]]
```

```
{{x → -1}, {x → 1}}
```

```
-2 + 2 x + 2 x^2 - 2 x^3
```

```
Double Integral:  $-\frac{8}{3}$ 
```

ii)

```

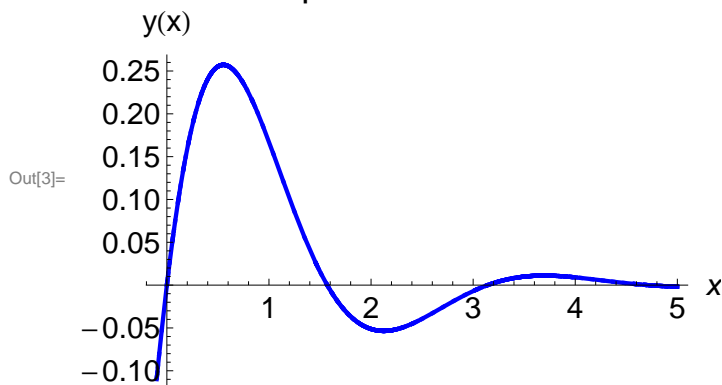
In[1]:= ClearAll[x, y];
DSolve[y''[x] + 2 y'[x] + 5 y[x] == 0, y[x], x]
DSolve[{y''[x] + 2 y'[x] + 5 y[x] == 0,
  y'[0] == 1, y[0] == 0}, y[x], x]
fgr = Plot[ $\frac{1}{2} e^{-x} \sin[2 x]$ , {x, -0.1, 5},
  PlotStyle -> Thick, ColorFunction -> Function[Blue],
  AxesLabel -> {x, "y(x)"},
  BaseStyle -> {FontFamily -> "Arial", FontSize -> 14},
  PlotRange -> All, AxesOrigin -> {0, 0},
  PlotLabel -> "partial solution"]

```

Out[1]= $\{\{y[x] \rightarrow e^{-x} C[2] \cos[2 x] + e^{-x} C[1] \sin[2 x]\}\}$

Out[2]= $\left\{\left\{y[x] \rightarrow \frac{1}{2} e^{-x} \sin[2 x]\right\}\right\}$

partial solution



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i)

```
f[x_, y_] := x^3 - 12 x + y^2 + 1
```

```
D[f[x, y], x]
```

```
D[f[x, y], y]
```

```
Solve[{D[f[x, y], x] == 0, D[f[x, y], y] == 0}, {x, y}]
```

```
-12 + 3 x^2
```

```
2 y
```

```
{{x -> -2, y -> 0}, {x -> 2, y -> 0}}
```

```
A = D[D[f[x, y], x], x] /. {x -> -2, y -> 0};
```

```
B = D[D[f[x, y], x], y] /. {x -> -2, y -> 0};
```

```
C1 = D[D[f[x, y], y], y] /. {x -> -2, y -> 0};
```

```
z = A * C1 - B^2; Print["D = ", z, " , ", "A = ", A]
```

```
D = -24 , A = -12
```

```
D < 0 saddle point
```

```
A = D[D[f[x, y], x], x] /. {x -> 2, y -> 0};
```

```
B = D[D[f[x, y], x], y] /. {x -> 2, y -> 0};
```

```
C1 = D[D[f[x, y], y], y] /. {x -> 2, y -> 0};
```

```
z = A * C1 - B^2; Print["D = ", z, " , ", "A = ", A]
```

```
D = 24 , A = 12
```

```
D > 0 and A > 0 minimum
```

```
Plot3D[f[x, y], {x, -3, 3}, {y, -3, 3},
```

```
  AxesLabel -> {"x", "y", "f(x,y) "},
```

```
  BaseStyle -> {FontFamily -> "Arial", FontSize -> 12}]
```

ii)

```
ClearAll[x, y, z, f];  
<< VectorAnalysis`;  
SetCoordinates[Cartesian[x, y, z]];  
F[x_, y_, z_] := Grad[f]  
Print["Curl F = ", Curl[F[x, y, z]]]
```

```
Curl F = {0, 0, 0}
```