18 September 2015

1 o

i)

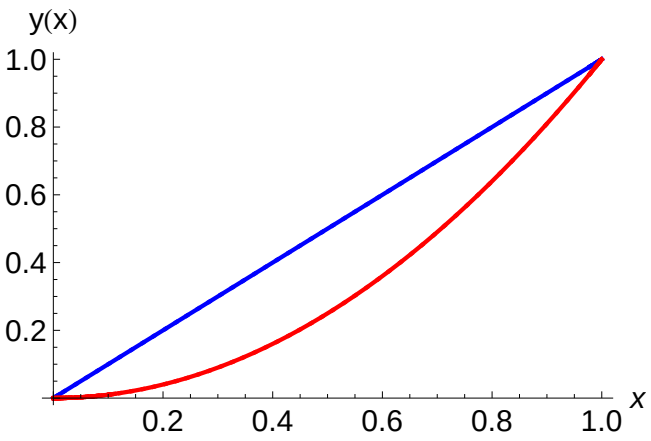
```
ClearAll[x, y, z];
<< VectorAnalysis‘;
SetCoordinates[Cartesian[x, y, z]];
F[x_, y_, z_] :=
 {4 x^3 y^2 z^3, 2 x^4 y z^3, 3 x^4 y^2 z^2}
Curl[F[x, y, z]]
ϕ[x_, y_, z_] := x^4 y^2 z^3
Print["Line Integral : ", ϕ[1, -1, 3] - ϕ[-1, 1, -2]]
```

```
{0, 0, 0}
Line Integral : 35
```

2 o

i)

```
fgr1 = Plot[x, {x, 0, 1}, PlotStyle → Thick,
    ColorFunction -> Function[Blue]];
fgr2 = Plot[x^2, {x, 0, 1}, PlotStyle → Thick,
    ColorFunction -> Function[Red]];
fgr = Show[fgr1, fgr2, PlotRange → All,
   AxesLabel → {x, "y(x)"},
   BaseStyle → {FontFamily → "Arial", FontSize → 14},
   AxesOrigin → {0, 0}]
Solve[x == x^2, x]
Integrate[x y, {y, x^2, x}]
Print["Double Integral: ",
 Integrate[x y, {x, 0, 1}, {y, x^2, x}]]
```



$$\{\{x \to 0\}, \{x \to 1\}\}$$

$$\frac{x^3}{2} - \frac{x^5}{2}$$

Double Integral: $\frac{1}{24}$

ii)

```
f[x_, y_] := x^2 + 2 y^2 - 2 x - 4 y + 1
D[f[x, y], x]
D[f[x, y], y]
Print["Critical Point = ",
 Solve[{D[f[x, y], x] == 0, D[f[x, y], y] == 0}, {x, y}]]
```

```
-2 + 2 x
-4 + 4 y
```

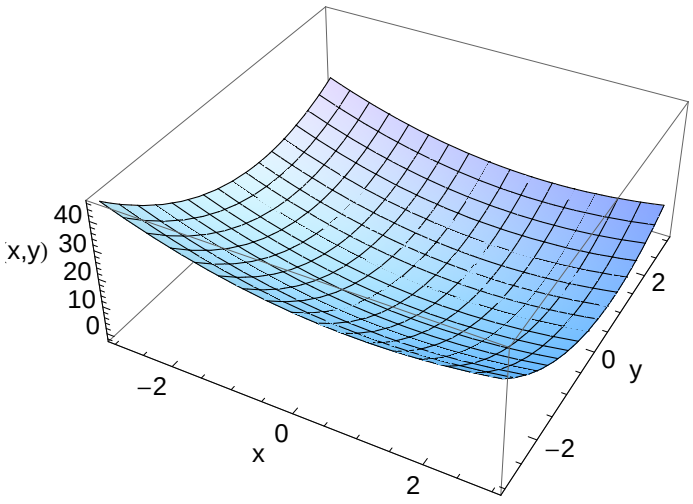
```
Critical Point = {{x → 1, y → 1}}
```

```
A = D[D[f[x, y], x], x] /. {x → 1, y → 1};
B = D[D[f[x, y], x], y] /. {x → 1, y → 1};
C1 = D[D[f[x, y], y], y] /. {x → 1, y → 1};
z = A * C1 - B^2; Print["D = ", z, "  ,  ", "A = ", A]
```

```
D = 8  ,  A = 2
```

D > 0 and A > 0 minimum

```
Plot3D[f[x, y], {x, -3, 3}, {y, -3, 3},
 AxesLabel → {"x", "y", "f(x,y)  "},
 BaseStyle → {FontFamily → "Arial", FontSize → 12}]
```



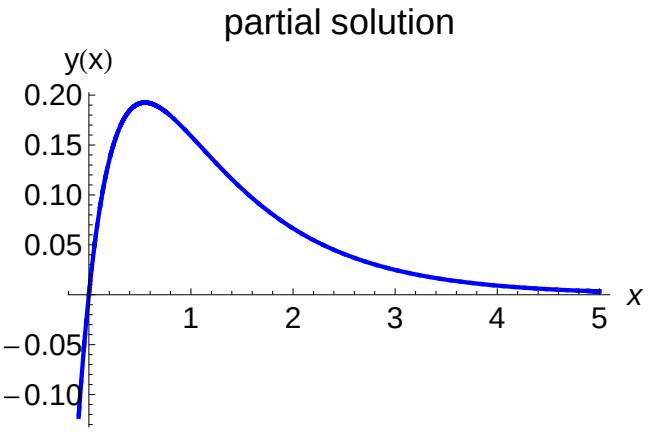
3 o

i)

```
ClearAll[x, y];
DSolve[y''[x] + 4 y'[x] + 3 y[x] == 0, y[x], x]
DSolve[{y''[x] + 4 y'[x] + 3 y[x] == 0,
  y'[0] == 1, y[0] == 0}, y[x], x]
fgr = Plot[1/2 e^(-3 x) (-1 + e^(2 x)), {x, -0.1, 5},
  PlotStyle → Thick, ColorFunction -> Function[Blue],
  AxesLabel → {x, "y(x)"},
  BaseStyle → {FontFamily → "Arial", FontSize → 14},
  PlotRange → All, AxesOrigin → {0, 0},
  PlotLabel → "partial solution"]
```

$$\left\{\left\{y[x] \to e^{-3x} C[1] + e^{-x} C[2]\right\}\right\}$$

$$\left\{\left\{y[x] \to \frac{1}{2} e^{-3x} \left(-1 + e^{2x}\right)\right\}\right\}$$



ii)

```
ClearAll[x, y, z, f];
<< VectorAnalysis`;
SetCoordinates[Cartesian[x, y, z]];
r[x_, y_, z_] := Sqrt[x^2 + y^2 + z^2]
Print["Grad r = ", Grad[r[x, y, z]]]
```

$$\text{Grad r} = \left\{\frac{x}{\sqrt{x^2+y^2+z^2}}, \frac{y}{\sqrt{x^2+y^2+z^2}}, \frac{z}{\sqrt{x^2+y^2+z^2}}\right\}$$