17 June 2016

1 o

i)

```
Clear[x]; Clear[y]; Clear[z]
<< VectorAnalysis‘;
SetCoordinates[Cartesian[x, y, z]];
ClearAll[x, y, z];
f[x_, y_, z_] := Log[x^2 + y^2]
Print["grad f = ", Grad[f[x, y, z]]]
Print["Laplacian f = ",
 Simplify[Laplacian[f[x, y, z]]]
```

$$\text{grad f} = \left\{\frac{2\,x}{x^2+y^2}, \frac{2\,y}{x^2+y^2}, 0\right\}$$

$$\text{Laplacian f} = 0$$

ii)

```
Clear[x]; Clear[y]; Clear[z]; Clear[t]
x1 = 1; y1 = 0; z1 = 1; x2 = 2; y2 = 1; z2 = 4;
x[t_] := t x2 + (1 - t) x1
y[t_] := t y2 + (1 - t) y1
z[t_] := t z2 + (1 - t) z1
Print["x(t) = ", x[t]]
Print["y(t) = ", y[t]]
Print["z(t) = ", z[t]]
xd[t_] := D[x[t], t]
yd[t_] := D[y[t], t]
zd[t_] := D[z[t], t]
Print["x'(t) = ", xd[t]]
Print["y'(t) = ", yd[t]]
Print["z'(t) = ", zd[t]]
P[t_] := x[t]
Q[t_] := x[t] - y[t]
R[t_] := z[t]
Print["P(t) = ", Simplify[P[t]]]
Print["Q(t) = ", Simplify[Q[t]]]
Print["R(t) = ", Simplify[R[t]]]
Print["P(t)x'(t)+Q(t)y'(t)+R(t)z' =",
 Simplify[P[t] xd[t] + Q[t] yd[t] + R[t] zd[t]]]
w = Integrate[P[t] xd[t] + Q[t] yd[t] + R[t] zd[t],
   {t, 0, 1}];
Print["Linear Integral: ", w]
```

$$x(t) = 1 + t$$

$$y(t) = t$$

$$z(t) = 1 + 3\,t$$

$$x'(t) = 1$$

$$y'(t) = 1$$

$$z'(t) = 3$$

$$P(t) = 1 + t$$

$$Q(t) = 1$$

$$R(t) = 1 + 3\,t$$

$$P(t)x'(t) + Q(t)y'(t) + R(t)z' = 5 + 10\,t$$

Linear Integral: 10

2 o

i)

```
Integrate[x + y, {y, x, 2 x}]
Print["Double Integral: ",
 Integrate[x + y, {x, 0, 1}, {y, x, 2 x}]]
```

$$\frac{5\,x^2}{2}$$

Double Integral: $\frac{5}{6}$

ii)

```
f[x_, y_] := Exp[-x^2 - y^2]
D[f[x, y], x]
D[f[x, y], y]
Print["Critical point : ",
 Solve[{D[f[x, y], x] == 0, D[f[x, y], y] == 0}, {x, y}]]
```

$$-2\,e^{-x^2-y^2}\,x$$

$$-2\,e^{-x^2-y^2}\,y$$

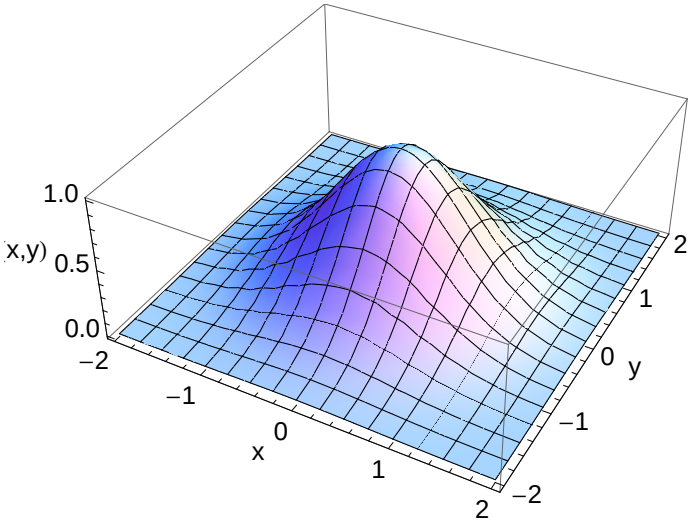
Critical point : $\{\{x \to 0, y \to 0\}\}$

```
A = D[D[f[x, y], x], x] /. {x → 0, y → 0};
B = D[D[f[x, y], x], y] /. {x → 0, y → 0};
C1 = D[D[f[x, y], y], y] /. {x → 0, y → 0};
z = A * C1 - B^2;
Print["D = ", z, "  ,  ", "A = ", A]
```

D = 4 , A = −2

D > 0 and A < 0 maximum

```
Plot3D[f[x, y], {x, -2, 2}, {y, -2, 2},
 AxesLabel → {"x", "y", "f(x,y)  "},
 BaseStyle → {FontFamily → "Arial", FontSize → 12}]
```



3 o

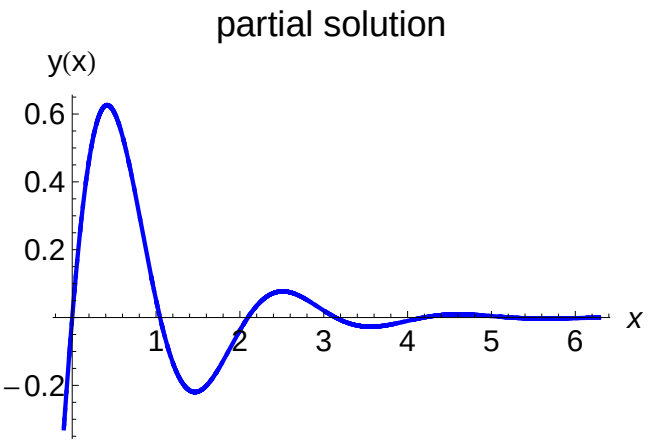
i)

```
Clear[y]; Clear[x]
DSolve[y''[x] + 2 y'[x] + 10 y[x] == 0, y[x], x]
DSolve[{y''[x] + 2 y'[x] + 10 y[x] == 0,
  y'[0] == 1, y[0] == 0}, y[x], x]
```

$$\{\{y[x] \to e^{-x} C[2] \cos[3x] + e^{-x} C[1] \sin[3x]\}\}$$

$$\left\{\left\{y[x] \to \frac{1}{3} e^{-x} \sin[3x]\right\}\right\}$$

```
fgr = Plot[e^-x Sin[3 x], {x, -0.1, 2 Pi},
  PlotStyle → Thick, ColorFunction -> Function[Blue],
  AxesLabel → {x, "y(x)"},
  BaseStyle → {FontFamily → "Arial", FontSize → 14},
  PlotRange → All, AxesOrigin → {0, 0},
  PlotLabel → "partial solution"]
```



ii)

```
G[s_] := (s + 1) / (s^2 + 4)
Print["Inverse Laplace transform : ",
 InverseLaplaceTransform[G[s], s, t]]
```

Inverse Laplace transform : $\frac{1}{2}(2\cos[2t] + \sin[2t])$