2 February 2016

1 o

i)

```
Clear[x]; Clear[y]; Clear[z]
<< VectorAnalysis‘;
SetCoordinates[Cartesian[x, y, z]];
ClearAll[x, y, z];
f[x_, y_, z_] := x^3 + y^2 z
Print["grad f = ", Grad[f[x, y, z]]]
Print["grad f |P = ",
 Grad[f[x, y, z]] /. {x → 1, y → 1, z → -2}]
```

$$\text{grad f} = \left\{3\,x^2,\ 2\,y\,z,\ y^2\right\}$$

$$\text{grad f}\,|P = \{3,\ -4,\ 1\}$$

ii)

```
Clear[x]; Clear[y]; Clear[z]; Clear[t]
x1 = 0; y1 = -1; z1 = 1; x2 = 2; y2 = 1; z2 = 3;
x[t_] := t x2 + (1 - t) x1
y[t_] := t y2 + (1 - t) y1
z[t_] := t z2 + (1 - t) z1
Print["x(t) = ", x[t]]
Print["y(t) = ", y[t]]
Print["z(t) = ", z[t]]
xd[t_] := D[x[t], t]
yd[t_] := D[y[t], t]
zd[t_] := D[z[t], t]
Print["x'(t) = ", xd[t]]
Print["y'(t) = ", yd[t]]
Print["z'(t) = ", yd[t]]
P[t_] := x[t] + y[t]
Q[t_] := x[t] - y[t]
R[t_] := z[t]
Print["P(t) = ", Simplify[P[t]]]
Print["Q(t) = ", Simplify[Q[t]]]
Print["R(t) = ", Simplify[R[t]]]
Print["P(t)x'(t)+Q(t)y'(t)+R(t)z' =",
 Simplify[P[t] xd[t] + Q[t] yd[t] + R[t] zd[t]]]
w = Integrate[P[t] xd[t] + Q[t] yd[t] + R[t] zd[t],
   {t, 0, 1}];
Print["Linear Integral: ", w]
```

$$x(t) = 2t$$

$$y(t) = -1 + 2t$$

$$z(t) = 1 + 2t$$

$$x'(t) = 2$$

$$y'(t) = 2$$

$$z'(t) = 2$$

$$P(t) = -1 + 4t$$

$$Q(t) = 1$$

$$R(t) = 1 + 2t$$

$$P(t)x'(t) + Q(t)y'(t) + R(t)z' = 2 + 12t$$

Linear Integral: 8

2 o

i)

```
Integrate[x - y, {y, x, 3 x}]
Print["Double Integral: ",
 Integrate[x - y, {x, 0, 1}, {y, x, 3 x}]]
```

$$-2 x^2$$

Double Integral: $-\frac{2}{3}$

ii)

```
f[x_, y_] := x^2 + 4 y^2 - 2 x - 8 y + 10
D[f[x, y], x]
D[f[x, y], y]
Solve[{D[f[x, y], x] == 0, D[f[x, y], y] == 0}, {x, y}]
```

$$-2 + 2 x$$

$$-8 + 8 y$$

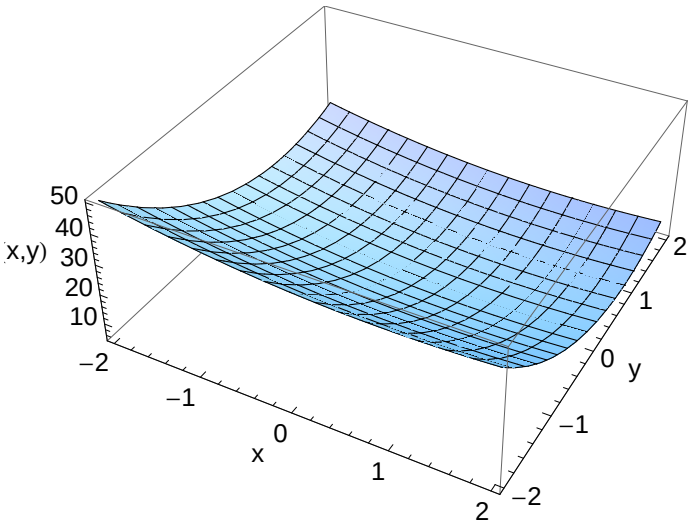
$$\{\{x \to 1, y \to 1\}\}$$

```
A = D[D[f[x, y], x], x] /. {x → 1, y → 1};
B = D[D[f[x, y], x], y] /. {x → 1, y → 1};
C1 = D[D[f[x, y], y], y] /. {x -> 1, y → 1};
z = A * C1 - B^2;
Print["D = ", z, "  ,  ", "A = ", A]
```

D = 16 , A = 2

D > 0 and A > 0 minimum

```
Plot3D[f[x, y], {x, -2, 2}, {y, -2, 2},
 AxesLabel → {"x", "y", "f(x,y)  "},
 BaseStyle → {FontFamily → "Arial", FontSize → 12}]
```



3 o

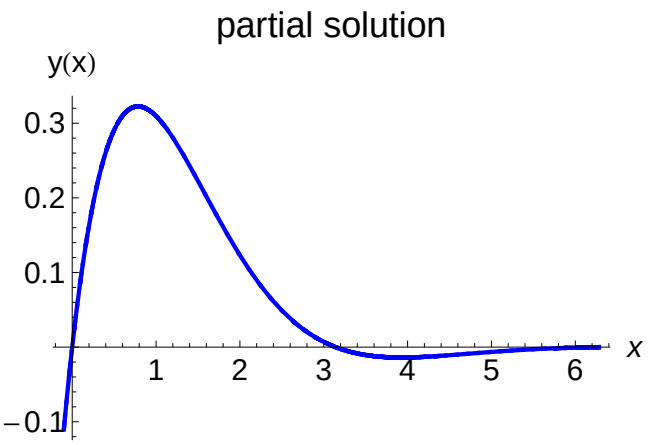
i)

```
Clear[y]; Clear[x]
DSolve[y''[x] + 2 y'[x] + 2 y[x] == 0, y[x], x]
DSolve[{y''[x] + 2 y'[x] + 2 y[x] == 0,
  y'[0] == 1, y[0] == 0}, y[x], x]
```

```
{{y[x] → ℯ^-x C[2] Cos[x] + ℯ^-x C[1] Sin[x]}}
```

```
{{y[x] → ℯ^-x Sin[x]}}
```

```
fgr = Plot[ℯ^-x Sin[x], {x, -0.1, 2 Pi},
  PlotStyle → Thick, ColorFunction -> Function[Blue],
  AxesLabel → {x, "y(x)"},
  BaseStyle → {FontFamily → "Arial", FontSize → 14},
  PlotRange → All, AxesOrigin → {0, 0},
  PlotLabel → "partial solution"]
```



ii)

In[8]:=
```
f[x_, y_] := Exp[-x^2 - y^2]
x0 = 1; y0 = -1;
f0 = f[x0, y0];
fx = D[f[x, y], x] /. {x → x0, y → y0};
fy = D[f[x, y], y] /. {x → x0, y → y0};
z = f0 + fx (x - x0) + fy (y - y0);
Print[
 "Equation of tangent plane: z = ", Simplify[z]]
```

Equation of tangent plane: $z = \frac{5 - 2x + 2y}{e^2}$