

11 September 2013

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i)

```
<< VectorAnalysis`;  
ClearAll[x, y, z]  
SetCoordinates[Cartesian[x, y, z]];  
ClearAll[x, y, z];  
 $\phi[x_, y_, z_] := x y z^2$ ;  
Print["potential  $\phi =$ ",  $\phi[x, y, z]$ ]  
Print["graf( $\phi$ ) = ", Grad[ $\phi[x, y, z]$ ]]  
Print["Linear Integral: ",  $\phi[2, 1, 2] - \phi[1, -1, 1]$ ]
```

potential  $\phi = x y z^2$

graf( $\phi$ ) =  $\{y z^2, x z^2, 2 x y z\}$

Linear Integral: 9

ii)

```
DSolve[y''[x] + 2 y'[x] + 10 y[x] == 0, y[x], x]
```

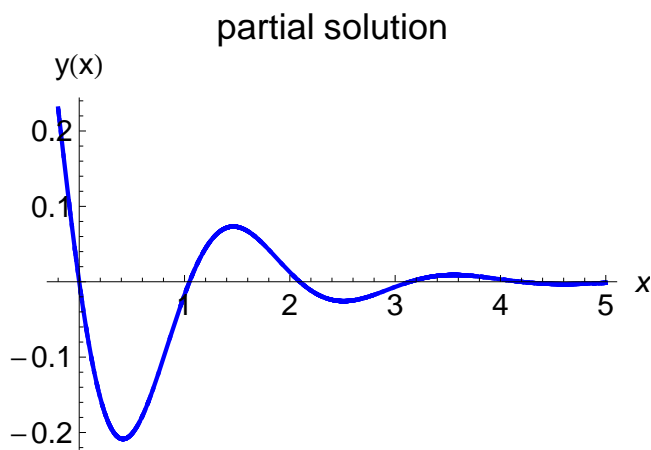
```
DSolve[{y''[x] + 2 y'[x] + 10 y[x] == 0,
```

```
  y'[0] == -1, y[0] == 0}, y[x], x]
```

```
{ {y[x]  $\rightarrow e^{-x} C[2] \text{Cos}[3 x] + e^{-x} C[1] \text{Sin}[3 x]$  } }
```

```
{ {y[x]  $\rightarrow -\frac{1}{3} e^{-x} \text{Sin}[3 x]$  } }
```

```
fgr = Plot[- $\frac{1}{3}$  e-x Sin[3 x], {x, -0.2, 5},
  PlotStyle -> Thick, ColorFunction -> Function[Blue],
  AxesLabel -> {x, "y(x)"},
  BaseStyle -> {FontFamily -> "Arial", FontSize -> 14},
  PlotRange -> All, AxesOrigin -> {0, 0},
  PlotLabel -> "partial solution"]
```



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i)

```
Integrate[x - y, {y, 0, Sqrt[x]}]
```

```
Print["Double Integral: ",
```

```
  Integrate[x - y, {x, 0, 1}, {y, 0, Sqrt[x]}]]
```

$$-\frac{x}{2} + x^{3/2}$$

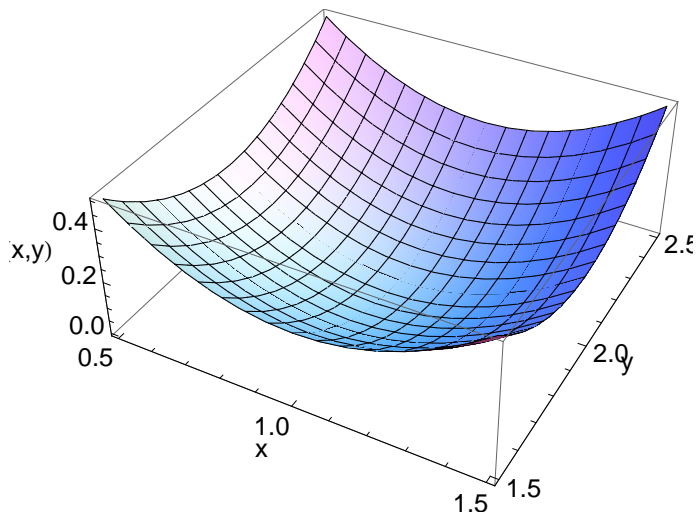
Double Integral:  $\frac{3}{20}$

ii)

```

f[x_, y_] := x^2 + y^2 - 2 x - 4 y + 5
D[f[x, y], x]
D[f[x, y], y]
Solve[{D[f[x, y], x] == 0, D[f[x, y], y] == 0}, {x, y}]
-2 + 2 x
-4 + 2 y
{{x -> 1, y -> 2}}
A = D[D[f[x, y], x], x] /. {x -> 1, y -> 2}
B = D[D[f[x, y], x], y] /. {x -> 1, y -> 2}
C1 = D[D[f[x, y], y], y] /. {x -> 1, y -> 2}
z = A * C1 - B^2
2
0
2
4
D > 0 and A > 0   minimum
Plot3D[f[x, y], {x, 0.5, 1.5}, {y, 1.5, 2.5},
  AxesLabel -> {"x", "y", "f(x,y)  "},
  BaseStyle -> {FontFamily -> "Arial", FontSize -> 12}]

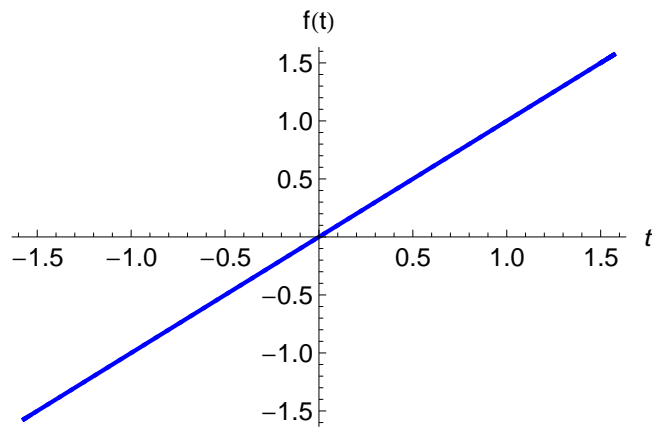
```



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i)

```
fgr = Plot[t, {t, -Pi / 2, Pi / 2},  
  PlotStyle -> Thick, ColorFunction -> Function[Blue],  
  AxesLabel -> {t, "f(t)"},  
  BaseStyle -> {FontFamily -> "Arial", FontSize -> 12}]
```



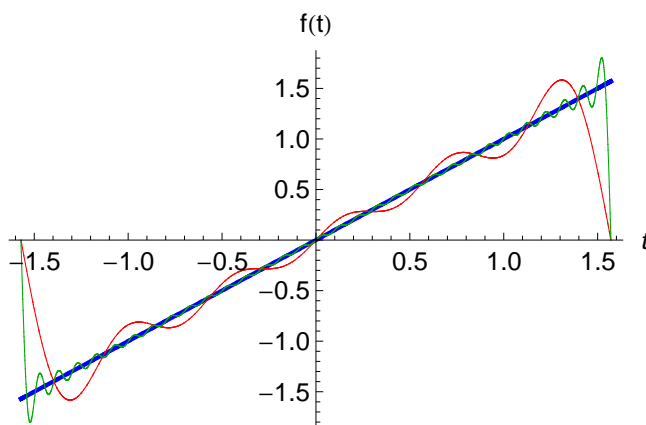
```

T = Pi;
bn = (4 / T) Integrate[t Sin[(2 n Pi t) / T], t]
bn = (4 / T)
      Integrate[t Sin[(2 n Pi t) / T], {t, 0, Pi / 2}] /.
      {Cos[n Pi] → (-1) ^ n, Sin[n Pi] → 0};
Print["b_n = ", bn]
Table[bn, {n, 1, 31}];
S5 = Sum[bn Sin[(2 n Pi t) / T], {n, 1, 5}];
S31 = Sum[bn Sin[(2 n Pi t) / T], {n, 1, 31}];
gf5 = Plot[S5, {t, -Pi / 2, Pi / 2},
      PlotRange → All, ColorFunction → Function[Red]];
gf31 = Plot[S31, {t, -Pi / 2, Pi / 2}, PlotRange → All,
      ColorFunction → Function[Darker[Green]]];
fg = Show[fgr, gf5, gf31, PlotRange → All,
      BaseStyle → {FontFamily → "Arial", FontSize → 12}]

```

$$4 \frac{\left( -\frac{t \cos[2 n t]}{2 n} + \frac{\sin[2 n t]}{4 n^2} \right)}{\pi}$$

$$b_n = -\frac{(-1)^n}{n}$$



ii)

```

Print["g(t) = ",
      InverseLaplaceTransform[(s - 4) / (s ^ 2 + 4), s, t]]

```

$$g(t) = \cos[2 t] - 2 \sin[2 t]$$