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Separation between reference surfaces of selected vertical datums

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Abstract

This paper discusses the separation between the reference surface of several vertical datums and the geoid. The data used includes a set of Doppler positioned stations, transformation parameters to convert the Doppler positions to ITRF90, and a potential coefficient model composed of the JGM-2 (NASA model) from degree 2 to 70 plus the OSU91A model from degree 71 to 360. The basic method of analysis is the comparison of a geometric geoid undulation derived from an ellipsoidal height and an orthometric height with the undulation computed from the potential coefficient model The mean difference can imply a bias of the datum reference surface with respect to the geoid. Vertical datums in the following countries were considered: England, Germany, United States, and Australia. The following numbers represent the bias values of each datum after adopting an equatorial radius of 6378136.3m: England (-87 cm), Germany (4 cm), United States (NGVD29 (-26 cm)), NAVD88 (-72 cm), Australia AHD (mainland, -68 cm); AHD (Tasmania, -98 cm). A negative sign indicates the datum reference surface is below the geoid. The 91 cm difference between the datums in England and Germany has been independently estimated as 80 cm. The 30 cm difference between AHD (mainland) and AHD (Tasmania) has been independently estimated as 40 cm. These bias values have been estimated from data where the geometric/ gravimetric geoid undulation difference standard deviation, at one station, is typically ±100 cm, although the mean difference is determined more accurately.

The results of this paper can be improved and expanded with more accurate geocentric station positions, more accurate and consistent heights with respect to the local vertical datum, and a more accurate gravity field for the Earth. The ideas developed here provide insight on the determination of a world height system.

Introduction

This paper is written to estimate the linear separation between the origin reference surface of several vertical datums. The process for doing this inherently implies a concept of a world height system as discussed by Rapp and Balasubramania (1992). This paper is not meant to be an extensive analysis of theoretical procedures involved in vertical datum definition and connection. Recent papers in this area include those by Xu and Rummel (1991) and Heck and Rummel (1990). This paper is not meant to show results of the latest measurement procedures. Examples of such papers are those of Rizos, Coleman and Ananga (1991) and Pan and Sjöberg (1993). This paper is designed to examine some earlier ideas (Rapp, 1983) using the latest gravity field models and terrestrial reference frame transformations.

Principles

Let N be the geoid undulation referred to a geocentric reference frame, an ellipsoid of a specified flattening, and an equatorial radius considered to be an optimum estimate. (Optimum may be a matter of opinion so this term is loosely used here.) Our calculation of N will be done through a potential coefficient model of degree L. Given the fully normalized coefficients C_{nm}, S_{nm}, one has

$$N(r,\theta,\lambda) = \frac{GM}{\gamma r}\sum_{n=2}^{L}\left(\frac{a}{r}\right)^{n} \cdot \sum_{m=0}^{n}(C_{nm}\cos m\lambda + S_{nm}\sin m\lambda)P_{nm}(\cos\theta) + c \tag{1}$$

where the usual definitions (Rapp and Pavlis, 1989) apply. The c term represents the downward continuation effects from an evaluation of the series at a valid point in space to a point on the geoid. We will assume that c is zero in this analysis. The even degree zonal harmonics in (1) refer to the adopted reference ellipsoid.

Let h be the ellipsoidal height of a point computed from rectangular coordinates in a geocentric reference frame in a true scale system. Let H be the orthometric height with respect to the geoid that is described by the N values of eq. (1). Then:

$$h = H + N \quad (2)$$

If one were given h and N then H can be computed. The accuracy of the determination of this H will depend on the accuracy of h, N and the optimum equatorial radius.

Consider next an orthometric height (H_D) referred to a specific vertical datum D. The ideal situation is that the vertical datum is defined by an equipotential surface, near mean sea level, at one specific point. In practice, vertical datums may have been defined by fixing mean sea level to zero orthometric elevation. In this case, the reference surface and the H values have some distortion with respect to the ideal case. In this paper, we will assume that a vertical datum is defined by a unique reference surface. This surface may not and actually will not coincide with the ideal reference surface, the geoid.

Let B be the bias between the ideal system and a specific system such that:

$$H = H_D + B \quad (3)$$

where H is the orthometric height in the ideal system and H_D is the corresponding height in the datum system. A positive B indicates the specific datum reference surface is above the ideal reference surface. We can estimate B by substituting (3) into (2):

$$B = (h - H_D) - N \quad (4)$$

The evaluation of B should be done using numerous stations connected to the datum D. Averaging individual values can reduce the noise inherent in the estimate of the three quantities on the right hand side of eq. (4). A comparison of B from different datums will yield information on the relative positions of these vertical datums.

Data

The stations to be analyzed for this paper have been positioned through Doppler observation techniques during the time period 1971-1986. A description of some of the processing techniques used in the estimation of the precise positions of an analogous station set may be found in Weigel (1993, p. 14). With this station set, "heights" above mean sea level are generally available. These heights are not precisely defined in terms of datum reference, type of orthometric height, or if in fact they may be normal heights. Although definitive results of this type of investigation require such information, this paper is considered a demonstration study and we will assume that the mean sea level heights are orthometric heights connected to the vertical datum of the region or country. Ultimately, we will restrict our analysis to stations that can be associated with a specific vertical datum.

Before any analysis is carried out, the ellipsoidal height given in the original data record is transferred to the ITRF90 system (Boucher and Altamimi, 1992, Table 4 (NWL 9D)) and referred to an ellipsoid whose equatorial radius is 6378136.3m and flattening is 1/298.257.

Analyses such as these are made more feasible as the knowledge of the Earth's gravity field improves. One of the recent models is JGM-2 (Lerch et al., 1993) which is a combination model complete to degree 70. This model has been augmented by the OSU91A model (Rapp, Wang, Pavlis, 1991) from degree 71 to degree 360. Although all tests to be described in this paper have been carried out with both the OSU91A and JGM-2 (augmented) model only (with one exception) results with the most current model (JGM-2) will be given. The geoid undulation can be computed from this data with an estimated accuracy of ±57 cm (Rapp, Wang, Pavlis, p. 64, ibid.) globally, which may be considerably poorer in areas lacking adequate gravity information.

Results - Global

The first computation involved the examination of all Doppler positions independent of their geographic location. Undulation comparisons were made with both the OSU91A and JGM-2 (augmented) potential coefficient models. In this analysis, stations with residuals exceeding 2m in absolute value were rejected from the statistical computations. This 2m criteria is tighter than the 4m criteria used in past studies, but it is consistent with the criteria adopted for the regional studies to be reported shortly. The 2m criteria is basically twice the standard deviation of the geometric/gravimetric difference and thus is a 2 sigma criteria. The statistics on the differences are given in Table 1 where the mean difference is defined through the geometric undulation minus the potential coefficient implied undulation. The total number of stations considered was 2033. The results indicate little preference for one geopotential model although the use of the 91A model allows 11 more stations to be considered. If the comparisons were made being more selective on Doppler stations to be used (at least 35 passes are observed after June 1977) the mean differences given in Table 1 become more positive by 15 cm and the standard deviation decreases to ±87 cm. Assuming, somewhat incorrectly, that the undulation differences are independent, the standard deviation of the mean difference is approximately ±3 cm.

Table 1

Mean and Standard Deviation of the Geometric and Gravimetric Undulation Difference for a Global Set of Doppler Stations

	Geopotential Model	
	OSU91A	JGM-2/OSU91A
Mean Diff.	-9 cm	-15 cm
SD Diff.	±100 cm	99 cm
No. Stat. Accepted	1428	1417

Results - England/Germany

The next step was to examine stations that could be closely associated with two regions with different vertical datums. The first area was England with stations selected in the region 50° to 60° N, 355° to 2°E. We assume that the orthometric heights we have are given in the Newlyn Datum

(Ashkenazi et al., 1990). The second area was Germany with stations selected in the region 48° to 54° N, 6° to 13°E. We assume that the orthometric heights we have refer to the vertical datum described in Torge (1980, Section 6.2.3).

The results of the comparisons, based on the 2m rejection criteria, are given in Table 2. The mean differences imply that the Newlyn Datum is -87 cm below the geoid while the German datum is 4 cm above the geoid. The difference implies that the German vertical datum reference surface is 91 cm above the Newlyn Datum.

Table 2

Mean and Standard Deviation of the Geometric and Gravimetric Undulation Difference for Doppler Stations in England and Germany

	England	Germany
Mean Diff.	-87 cm	4 cm
SD Diff.	±64 cm	±75 cm
No. Stat. Accepted	29	28

An independent estimate of the above can be obtained as follows. Willis et al (1989) estimate (on the basis of several sources of information) that the origin of the French vertical datum (IGN69) is 30 cm ±8 cm above the origin surface of the Newlyn Datum (ODN). On the basis of the comparison of heights near the French/German border (Boucher 1993, private communication), the German system origin is 50 cm above the IGN69 system (i.e. H (Germany) = H (IGN69) - 50 cm). This numerical value can be caused not only by vertical datum differences, but also differences in normal heights and orthometric heights. The latter we will ignore for this paper. Taking the 30 and 50 cm into account would imply that the German origin surface is 80 cm above the Newlyn Datum surface. This number can be contrasted with the 91 cm obtained through the Doppler station analysis.

Results in the United States

The orthometric heights for the Doppler stations in the United States are given with respect to the National Geodetic Vertical Datum of 1929 (NGDV29). This datum was defined through a general adjustment in 1929 with height constraints imposed at 26 tide gauge stations. The orthometric heights were computed using normal corrections since no gravity observations were available along the leveling lines.

A new adjustment of leveling data has led to the North American Vertical Datum of 1988 (Zilkoski, Richards, and Young, 1992). This datum is based on geopotential numbers fixing the height at a primary tidal benchmark. Although we did not have heights for the Doppler stations directly in the NAVD88 system, a datum conversion software program (VERTCON, version 1.0, February 1993) has been developed at the National Geodetic Survey. This software was used to convert the NGVD29 heights into NAVD88 heights using a transformation procedure in lieu of the direct adjustment of the points in NAVD88.

The stations to be used in this analysis were initially selected in the region: 30° to 45°N, 240° to 290°E. This station set was then thinned to eliminate stations geographically close so that a file containing 350 stations was analyzed. The results of the undulation comparison with the JGM-2/OSU91A potential coefficient model and a 2m rejection criteria are given in Table 3.

Table 3

Mean and Standard Deviation of the Geometric and Gravimetric Undulation Difference for Doppler Stations in the United States

	Vertical Datum	
	NGVD29	NAVD88
Mean Diff.	-26 cm	-72 cm
SD Diff.	±83 cm	±100 cm
No. Stations	321	321

The mean differences for NGVD29 and NAVD88 differ by 46 cm. Somewhat surprising is the somewhat poorer fit (100 cm vs 83 cm) when the NAVD88 datum is used. One would suspect that the NAVD88 heights should be more consistent with the ellipsoidal heights/gravimetric undulations than the NGVD29 heights which are given in a distorted datum.

Figure 4 of (ibid.) show the height differences between NAVD88 and NGVD29. This map shows that the heights with respect to the two datums are quite consistent in the eastern half of the country. Due to this, additional tests were made restricting the analysis to stations east of 254°E. The results of this analysis are given in Table 4. We see a substantial change (from Table 3) of the mean difference for stations on NAVD88 (-72 cm (Table 3) to -38 cm (Table 4)). The corresponding change in the NGVD29 difference is only 8 cm. The standard deviation of the undulation differences in going from NGVD29 to NAVD88 is smaller than the corresponding case in Table 3.

Table 4

Mean and Standard Deviation of the Geometric and Gravimetric Undulation Difference for Doppler Stations in the Eastern United States

	Vertical Datum	
	NGVD29	NAVD88
Mean Diff.	-18 cm	-38 cm
SD Diff.	±86 cm	±92 cm
No. Stations	180	180

These computations were repeated restricting the analysis to stations where the number of satellite passes was greater than 35 and the observation data was after June 15, 1978. The standard deviations would typically be reduced by about 10%. The mean differences would change by 7 to 12 cm.

The results of Table 3 make it unclear as to the appropriate mean difference to be associated with the two vertical datums. One can argue that a single parameter can not be used to represent the difference between the NGVD29 reference surface and the geoid because of the manner in which NGVD29 was adjusted. However, there is only a 8 cm difference between the total station set and the eastern region data. On the other hand, one would expect the greatest consistency with the NAVD88 system. Unfortunately the mean difference differs by 34 cm from the total station set in the eastern station set. With no other information available, one perhaps should use the results given in Table 3 understanding the value for NAVD88 is subject to change when more accurate heights in the NAVD88 system are used in the analysis. Specifically from Table 3 one estimates that the NGVD29 reference surface is located 26 cm below the geoid while the NAVD88 reference surface is 72 cm below the geoid.

Results in Australia

The heights available at the Doppler stations are approximate orthometric heights referred to the Australian Height Datum (AHD). The development of this datum is described by Leppert (1974) and Rizos et al. (1991). The levelling adjustment was carried out in 1971 using height constraints at 30 tide gauge stations around the coastline of Australia.

The Doppler stations were well distributed around Australia. The results of the analysis are given in Table 5 (AHD (M)). The mean difference of -68 cm suggests that the average AHD reference surface is located 68 cm below the geoid as defined by the JGM-2/OSU91A potential coefficient model. This magnitude conflicts with estimates of dynamic topography which suggests that mean sea level in the vicinity of Australia is above the geoid. (For example, see Figure 11 in Rapp, Wang, Pavlis (1991)).

Table 5

Mean and Standard Deviation of the Geometric and Gravimetric Undulation Difference for Doppler Stations in Australia and Tasmania

	AHD (M)	AHD (T)
Mean Diff.	-68 cm	-98 cm
SD Diff.	96	66
No. Stations	85	4

An attempt was made to determine the difference between the AHD as used in the mainland and the AHD used in Tasmania. This test was a follow-up to the study described by Rizos, Coleman and Ananga (1991). The first step was to use four Doppler stations in Tasmania that yielded a mean difference of -98 cm with a standard deviation of ±66 cm (see Table 5 AHD(T)). This result implies that the AHD on the mainland has its reference surface 30 cm higher than the AHD in Tasmania. This value is similar to an updated (from Rizos, Coleman, Ananga (1991)) estimate of 40 cm (Coleman, 1993, private communication). A caveat here is that this analysis uses data distributed over all of Australia while the Rizos, Coleman, Ananga study considered the difference between the AHD (Victoria coast) and AHD (Tasmania).

Conclusions

This paper has demonstrated a procedure that estimates the separation between reference surfaces defined by several vertical datums and the geoid that is defined by geoid undulations computed from the JGM-2 (degree 2 to 70) and OSU91A (degree 71 to 360) potential coefficient model. The ellipsoidal heights of the stations are determined from Doppler positioning techniques after a transformation to the ITRF90 system. Although the accuracy of the vertical position derived from the Doppler positions is on the order of ±80 cm, the large number of stations on some vertical datums makes these tests meaningful.

In the analysis we analyzed data that can be associated with vertical datums in England, Germany, Australia, and the United States. We assume that the difference between the geoid and the reference surface of the vertical datum can be modeled by a single parameter. This would be appropriate if the datums were uniquely defined by a reference surface through a specified point. However, the adjustment of several of the datums was carried out by fixing heights at tide gauges distributed around the borders of the country. This fixing introduces a distortion in the reference surface (Laskowski, 1983) with respect to a single point related The reference surface. Our procedure then calculates a separation for some average reference surface representative of the area in which the stations are given.

The results for the differences in the origins of the vertical datums studied for this paper are shown in Figure 1. Not shown are the results for AHD (Tasmania) because of the few stations available. Two results for NAVD88 (U.S.) are shown because of the large differences in the result. One estimate is based on the complete station set after conversion from NGVD29 while the second estimate is based only on stations in the eastern part of the US. From Figure 1 one clearly sees the differences of the reference surfaces of a few vertical datums. It should be emphasized that the numerical value shown in tables and Figure 1 are dependent on many factors (station reference frame, geoid undulation model, equatorial radius, etc.).

The results (Table 3/4) for the two vertical datums in the U.S. are somewhat troubling. Why should the standard deviation of fit be poorer when the NAVD88 (a more rigorous datum) heights are used as opposed to the NGVD29 values? Why should the mean differences change so much (44 cm) when the results for the complete station set vs the eastern US station set of NAVD88 are considered? Additional study with more information on NAVD88 heights is needed.

Another area of concern is the magnitude of the geoid/datum separation for the Australian Height Datum. On the basis of oceanographic determinations of mean sea level one would expect the mean differences to be somewhat (~20 cm) positive instead of the -68 cm found in these results. On the encouraging side was the good agreement between the AHD(mainland)/AHD(Tasmania) difference between the 30 cm (AHD (M) higher)) and the recent estimate by Coleman of 40 cm.

To resolve some of these questions, more accurate computations are needed that would use more precise geocentric positions of stations on various vertical datums and more refined computation for geoid undulation values at the stations. Procedures to use this data for vertical datum definition and connections are described in Rapp and Balasubramania (1992).

And finally, the results of this study imply that height systems can be inconsistent by one or two meters. This is not new information as it is well known from studies of dynamic height variations in the oceans. There seems to be a clear need to develop a unique height system that can be tied to the ideal reference surface, the geoid. Given a sufficiently accurate representation of geoid undulations, orthometric heights can be derived from an ellipsoidal height computed from space positioning. Additional consideration is needed to define the type of orthometric height wanted, or to implement the determination of a normal height through a height anomaly calculation instead of a geoid undulation computation. Whatever the case, we are approaching, although not yet there, the situation of having sufficient gravity field information and ellipsoidal height values to define a world height system at the 50 - 100 cm accuracy level.

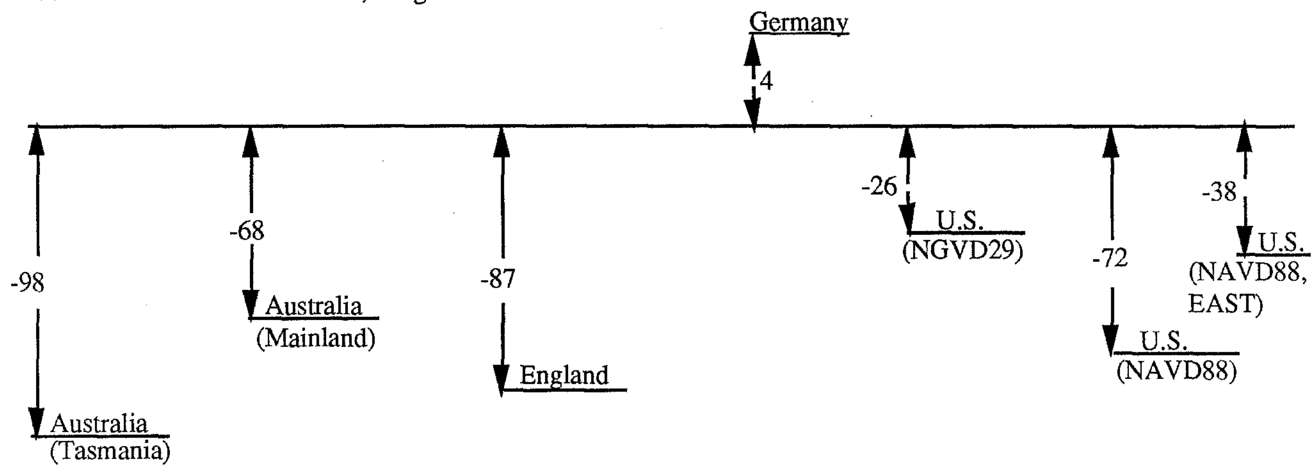

Fig. 1. Summary of Reference Surface Differences with Respect to the Geoid. Units are cm.

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