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THE ADJUSTMENT OF U.E.L.N. AS EXECUTED AT DELFT

I. COMPUTATIONS.

The work done at Delft can be divided into three parts :

1. The adjustment of the net;
2. The study of the precision of the net;
3. The comparison of the net with the mareograph observations.

The first and part of the second item have been explained in some detail in the preliminary report [1] (see references).

I.1 - The adjustment of the net.

The method employed at Delft is an adjustment by steps, as indicated by TIENSTRA, in which new ideas and methods developed after professor TIENSTRA's death have been applied.

The method is not the quickest or most economical one with respect to the computation of the corrections to be given to the observations. The reason for choosing an adjustment by steps was that this provides a means for comparing the different parts of the net, and that smaller matrix inversions are involved in the complete computation of the precision of the final result. A further advantage is that if a new part is added, the Yugoslavian net for instance, a change has to be made in the second step only.

The total net was split up in four partial nets, roughly as follows (see map) :

D. Denmark, Germany, the Netherlands, Belgium

C. Austria, Switzerland, Northern Italy

F. France

E. Spain and Portugal

This choice was made for several reasons.

Firstly, these nets have a convenient size with a view to the

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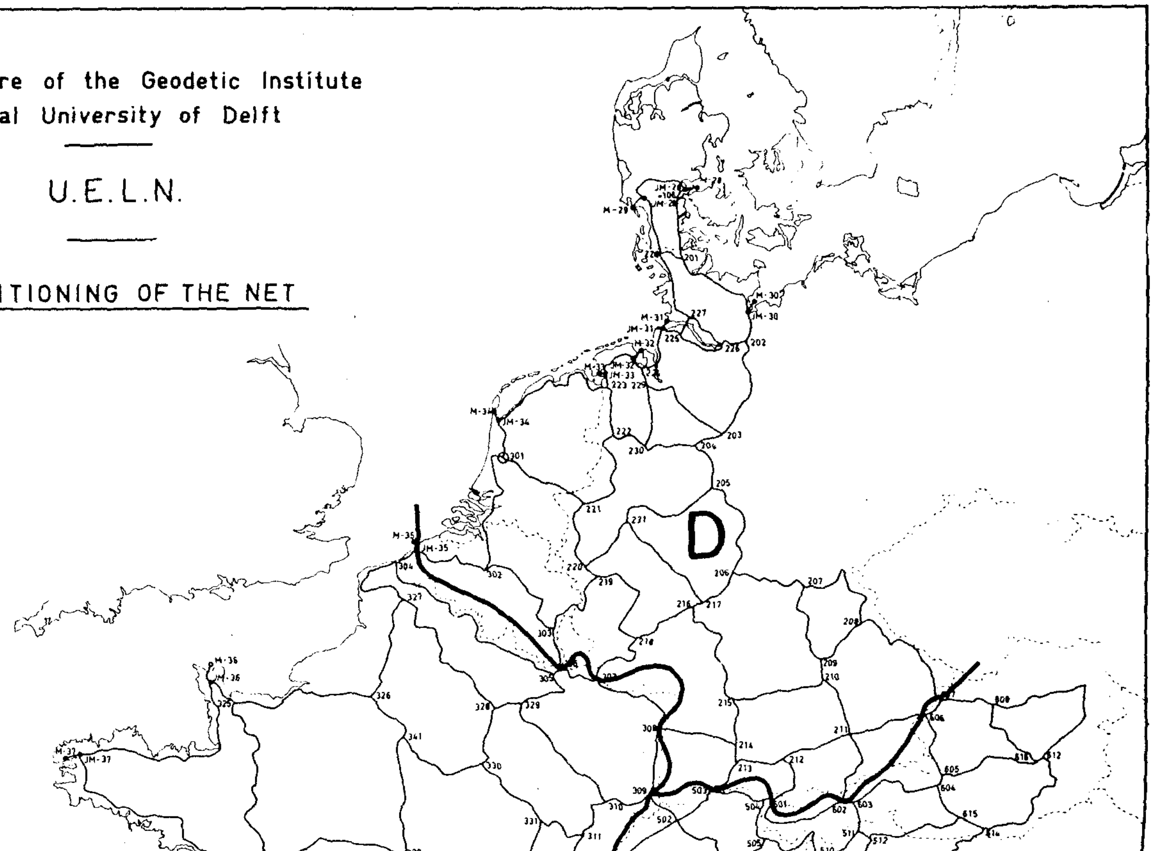
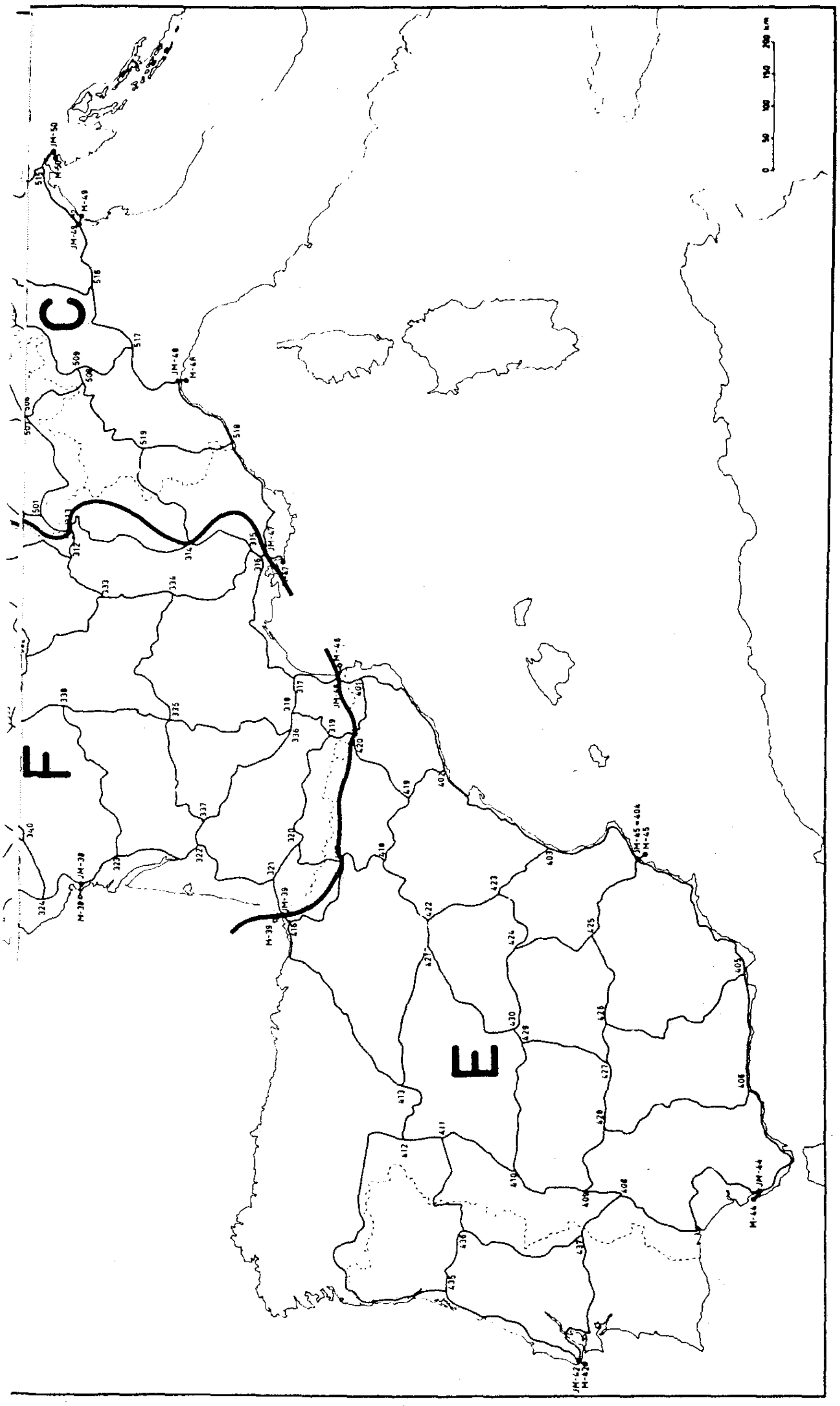
U.E.L.N.

PARTITIONING OF THE NET




C
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capacity of the computing machine that has been used;

Secondly, it was expected that these parts were fairly homogeneous as far as their accuracy is concerned;

Thirdly, the choice has been determined by geographical features.

Each partial net was adjusted in itself by the method of observation equations. The unknowns were the differences in geopotential number (g.p.n.) between a chosen datum point in each partial net and the other points in that net. The course of the computation is well-known and is given in matrix form in [1] , page 10.

The matrix of the normal equations for each partial net was inverted, so the result consisted of the values of the unknowns and their matrix of weight-coefficients, which is proportional to their variance-covariance matrix.

The second step was performed by the method of condition equations. The observations consisted on the one hand of the unknowns $\underline{x}$, computed in the previous step, and on the other hand of the corresponding differences $\underline{y}$ of geopotential numbers pertaining to points which had not been included in the first step, being the points at the edges of the partial nets, connected by still unadjusted levelling lines.

The great majority of unknowns from the first step have a coefficient zero in the condition equations; of course this does not mean that they do not get a correction from the second step of the adjustment. However, the second adjustment can, to begin with, be treated as if all these observations were not there, up to and including the computation of the correlates. The computation of their correction from the correlates follows exactly the same lines as for observations whose coefficient in all equations is not zero. The course of the computation is shown in the diagram in [1] page 10.

The corrections from first and second steps have to be added to obtain the final adjusted values. A number of simple additions then gives the differences of g.p.n. with respect to the datum point of the total net. The g.p.n.'s of the mareographs are found by some more additions.

The matrix of weight-coefficients of the final results is computed according to the well-known rules for the weight-coefficients of functions of adjusted observations.

Only a few remarks on the computation technique : the computations were carried out on an electronic computer of Dutch design, which is built in England, the ZEBRA.

The capacity of this relatively small but flexible computer was such that the inversion of the matrix of normal equations for each partial net could be done in one step. However, for the computation of the final matrix of weight-coefficients after the second step, the matrices to be multiplied were so big that partitions had to be made, which necessitated additional programming.

The computations were carried out with nine significant figures. One of the partial nets was adjusted by the method of condition equations and by the method of observation equations, to have a check on the programme and to study the effect of rounding-off. In both was included the computation of the weight-coefficients of adjusted heights; the results only differed up to 10 units of the last decimal.

Since the machine programme for this adjustment is now ready, there will be no difficulty in adjusting any other net in the same way. Therefore it is not necessary that the shape of the net is the same-this is important for a future repetition of the work. Anyhow, new observations in the net would have other weights than those used now so that any repetition of the observations would entail a complete new computation, which, however, is rendered easy by the programme.

I.2 - The precision of the net.

Each adjustment makes it possible to compute an estimate for the variance of an observation of unit weight, which we prefer to call an estimate for the variance factor.

The choice of the weight formula implies the choice of the variance factor, this factor is in this case 200. In our case, we could compute estimates for σ^2 from each partial net, from the whole first step of the adjustment, from the second step of the adjustment, and from the total adjustment.

If we denote the estimate by $\hat{\sigma}^2$, we have

$$\hat{\sigma}^2 = \frac{[pvv]}{\sigma} ,$$

where σ is the number of supernumerous observations. $\frac{\hat{\sigma}^2}{\sigma^2}$ has a known probability distribution, an $F_{\sigma,\infty}$ distribution, and by means of this distribution we have statistically compared the estimates of the variance factor found in the different partial nets and in the whole first step and in the second step of the adjustment. More details of these tests can be found in [1]. Only some results are given here :

In two of the four partial nets the estimate of σ^2 was, on the 5 % level of significance, significantly higher than 200. This was also the case in the first step as a whole, in the second step and in the adjustment as a whole. This might lead us to suspect that the observations were affected by systematic errors. But there is another type of F-test in which not one estimate is compared with the adopted theoretical value, but in which two estimates are compared with each other. This comparison has also been carried out and the result was that the differences between the estimates of σ^2 were not significant (two-sided tests on the 5 % level).

This may indicate that not model- or systematic errors are the cause of the significantly high values of $\hat{\sigma}^2$ found in nearly all previous tests, but that the overall level of the variance is higher than assumed. This would mean that some of the values for t^2 in the weight formulas are too small.

The power of the tests used has been evaluated, i.e. it has been computed what size a model error in a levelling line would have to attain in order to lead to a significantly too high $\hat{\sigma}^2$ (rejection of the null-hypothesis that no model-errors occur) with a probability of 80 %. It appeared that the smallest error in a levelling line that can be found by these tests is 2.5 cm. In certain parts of the net (for instance in France and Spain) model errors of up to 25 cm cannot be detected by these tests.

All weight-coefficients of the geopotential numbers in the whole net have been computed. This makes it possible to calculate the standard deviation of the difference in g.p.n. between any two points.

The following combinations were investigated :

a. All the points with respect to the Amsterdam datum,

b. All the combinations of two points directly connected by a levelling line,

c. The most important combinations of mareographs.

This research made clear, among other things, that the standard deviation of the points with respect to N.A.P. increases to 1.5 cm in Belgium, to 4 cm in North France, to 6 cm in South France and to 10 cm in Spain. Further the standard deviation increase to about 1 cm in Germany, 2 cm in Switzerland and Austria and 3 cm in North-Italy, and it grows to 2 cm in Denmark, 4 cm in South-Sweden, 7 cm in South-Norway, 20 cm in North-Sweden, and 24 cm in Finland and North-Norway.

These standard deviations differ slightly from the standard deviations obtained by the Finnish calculations, because we did not correct our weight-coefficients for land-uplift and because the used variance factors probably were not quite the same.

The standard deviation of the difference in height of two points connected by a levelling line, about 200 km in length, is in Denmark, Finland, Germany, the Netherlands and Portugal about 1 cm, in Austria, Italy and Switzerland about 1.5 cm, in Norway about 3 cm and in France and Spain about 4 cm.

The standard deviations of the differences in height of the mareographs are also very interesting. So we find a standard deviation of 3 cm between Denmark (M 28) and the Adriatic (M 49); of 5 cm between the North Sea (M 32) and the Mediterranean (M 47); of 6 cm between two mareographs (M 46, M 47) of the Mediterranean; of 20 cm between North-Sweden (M 13) and South-Sweden (M 15) and 25 cm between North-Norway (M 22) and South-Spain (M 44).

I.3 <u>The comparison of the net with the mareograph observations</u>.

The net furnishes geopotential numbers for mean sea level (M.S.L.) at the different mareographs. One may ask in general : are the differences in g.p.n. found due to M.S.L. not being an aequipotential surface or can they be ascribed to the inaccuracy of the net ? The investigation of this kind of questions has been done with the aid of statistical tests. One may set up a certain condition model by which hypothetical properties of M.S.L. are expressed. By adjusting the data furnished by the net to fit the conditions of the model, one gets an estimate for the variance factor, which can be tested by an F-test, to see to what extent the data fit the model.

A great difficulty in this approach was that only the probability distribution of the data furnished by the net was known. No data were available with regard to the accuracy and precision of the mareograph data, so that in the following it was assumed that all variance is produced by the net, and none by the mareographs. It must be borne in mind that a test can lead to rejection of a hypothesis, but cannot indicate exactly the cause of rejection. This calls for very great care in the interpretation of results.

Six different kinds of hypotheses were tested (one-sided tests on the 5 % level);

a. A mareographs has the height zero.
b. Two mareographs have the same height.
c. A group of mareographs have all the height zero.
d. A group of mareographs have all the same height.
e. The average height of a group of mareographs is zero.
f. Two groups of mareographs have the same average height.

The condition equations are respectively :

a. $h^i = 0$, for every mareograph apart.
b. $h^i - h^j = 0$, for several combinations of mareographs
c. $h^i = 0$, for a group of mareographs at the same time.
d. $h^i - h^G = 0$, for a group of G mareographs.
e. $h^G = 0$, for a group of G mareographs.
f. $h^{G_1} - h^{G_2} = 0$, for a combination of two groups of mareographs.

a. The first type of null-hypothesis leads only for 14 of the 44 mareographs to a significant difference. The most important results are, that the mareographs in the Gulf of Bothnia give <u>no</u> significant difference, and only the mareographs 34, 35, 48, 49 and 50 give a highly significant difference.

b. Investigations made in the Netherlands have shown that the datum point in Amsterdam does not quite correspond to M.S.L. and therefore the second type of our null-hypotheses is much more interesting than the first type. The most important significant differences appear with some combinations Gulf of Bothnia - North Sea (for instance 12 - 23, 15 - 17), with several combinations North Sea - Mediterranean (for instance 34 - 48, 38 - 48, 39 - 46) and only with one combination Gulf of Bothnia - Mediterranean (15 - 48). We have to remark that we did not investigate all possible combinations and also that not all tests are independent.

c. Up till now we have used in every test only one difference in height, but it is also possible to draw a group of mareographs together in our test. We set up then as many condition equations like (c) as there are mareographs in the group. We have formed three groups, namely A (the Atlantic), B (the Gulf of Bothnia) and M (the Mediterranean) with resp. 16, 15 and 6 mareographs.

The F-values and the critical values are resp. :

	F-value	critical value
A	12, 2	1, 8
B	2, 8	1, 8
M	25, 5	2, 2

Two things are conspicuous : In the first place that the testing of group A and M is very significant and in the second place that the testing of group B is also significant but not to such a high degree.

About group B we have to remark that the matrix of weight-coefficients is nearly singular. The near-singularity is caused by the fact that the whole Finnish net is connected with the central block only by one, for the greater part rather inaccurately measured levelling line. And the consequence of this near-singularity is that the inversion needed for the computation of the F-value, gives a very unreliable result. Another consequence of the near-singularity is that the F-value of group B is in fact determined only by the weight and the height of mareograph 15. The amounts in "[pvv]" belonging to the other mareographs all cancelled out, and so we did not test group B with respect to the datum point, but only mareograph 15.

d. The third kind of null-hypotheses has the same disadvantage as the first kind, namely that the datum point, whose height may not be M.S.L., is involved. Therefore we have computed an adjusted height h^G per group and so we come to our fourth kind of null-hypotheses.

We found for heights :

$$h^A = -\ 76 \text{ mm}$$
$$h^B = +\ 138 \text{ mm}$$
and $$h^M = -\ 298 \text{ mm}$$

The results of the tests are as follows :

	F-value	critical value
A	7, 9	1, 8
B	2, 2	1, 8
M	5, 0	2, 4

So we see again that our null-hypotheses are rejected, and that the significance of group B is doubtful. We also see that the differences of F-value and critical value are much smaller than with the null-hypotheses under item c. This suggests that our datum point does indeed not define M.S.L.

e. Not only the value of the adjusted height per group, but also its weight was computed, and this enables us to set up a fifth kind of null-hypotheses, namely that the adjusted heights of the three groups are zero. The results of these tests are as follows :

	F-value	critical value
A	77, 8	4, 0
B	10, 9	4, 0
M	128, 6	4, 0

We see that the null-hypotheses are again rejected, and for the Atlantic and Mediterranean with considerable significance. Anew the test of the adjusted height of the Gulf of Bothnia has little or no meaning, because the adjusted height is the equal to the height of mareograph 15, and so in fact we tested only that height.

f. The last kind of null-hypotheses tested was that the adjusted heights for a combination of two groups are the same. We give only the result of the testing of the null-hypotheses $h^A - h^M = 0$, because as shown above the adjusted height of the Gulf of Bothnia has no real sense. The result is as follows :

	F-value	critical value
A - M	75, 0	4, 0

Consequently this hypothesis has to be rejected also. These tests have an experimental character, because the setting up of a model requires a good knowledge of the physical phenomena in question; it belongs to the field of the oceanographers.

II. DISCUSSION.

A concurrence of circumstances in the Netherlands has been the cause that in recent years special attention has been directed to the interpretation of the results of survey measurements and adjustments, see [2] . The methods developed have been applied first to the field of cadastral and engineering surveys for which reference is made to [3] . The adjustment of U.E.L.N. offered an opportunity for experimentally applying the theory to a larger adjustment.

In part I it has been explained what computations and tests have been carried out, and it is meant here to discuss considerations and conclusions. It must be borne in mind that this experimental application to a large adjustment has the character of an improvisation because the complexity of the problem gave rise to some new aspects.

In studying the accuracy of the net, we have at the base of our investigation the step from the physical world in which the net has been measured to the mathematical world in which we carry out our computations. The part of the mathematical model which is expressed by the condition equations, is quite refined, as we deal with differences of geopotential numbers and so gravity is taken into account.

The stochastic model as expressed by the weights is probably also quite satisfactory for practical purposes. Its chief characteristics are that the observational quantities are assumed to have a correlation-free normal distribution, and that each country has given the value of t^2 for the observations it furnished; unfortunately, slightly different formulae have been used for the computation of t^2 . An investigation into the effect of these different formulae seems desirable. The net is inhomogeneous as to the age of its different parts. The partitioning of the net has made it possible to investigate its main parts separately, so that some sort of analysis could be made before getting all observations mixed in the adjustment.

The estimate of the variance factor affords a control on model- or systematic errors. It is remarkable that in the partial net F, which contains some old levelling lines, the estimate was not significantly too high so that it could not be concluded that systematic errors were present. Such systematic errors might also have arisen from the gravity values used. But because of the low weights assigned to the observations in question, these possible systematic errors have not made themselves felt.

In the partial nets D and E, the estimate σ^2 is significantly too high. The estimate resulting from the second step of the adjustment is also too high; this may be explained by the fact that the boundary lines between the partial nets in many cases ran across polygons with large misclosures, such as in the Ardennes, the Pyrenées, etc... Especially in these mountainous parts the misclosures may be due to systematic errors. But in view of the mutual testing of the partial nets and the testing of the partial nets together with the second step of the adjustment, it seems likely that this effect of systematic errors is not preponderant, and that a number of values of t^2 are too low.

The power of the tests has been investigated, i.e. it has been computed what size an error in a single levelling line must have, if it is to lead with 80 % probability to a significantly too high estimate of σ^2. We thus find for each levelling line the systematic error or blunder which can be detected by the tests used. The size of these errors is considerable, and may even attain 20 or 25 cm in certain parts. Now it is true that the levelling lines are all selected from much denser nets, so they are effectively controlled. But in [4], pages 150, 151, Dr. SIMONSEN has compared prof. KUKKAMAKI's and the present adjustment. In certain areas the accordance is not good because different lines were used; apart from the different methods of adjustment, this is an indication for the unreliability of the levellings.

To come back to the estimates of the variance factor : if the estimates from all partial nets had been scattered around the value 200, this value would have been adopted for computing the variances and covariances of the geopotential numbers of the nodal points. As it was, the estimate resulting from the whole adjustment, whose value was 405, had to be taken. If this had been the true variance factor it would be simple to find a 95 % confidence for the g.p.n. of each mareograph; this would have been an interval of about 3.92 times the standard deviation of the g.p.n. in question. Because the value 405 is only an estimate, the confidence intervals are somewhat more than 3.92 times the standard deviations.

After these general remarks on the net we shall try to draw some conclusions from the comparison with the mareographs.

In spite of the high quality of the Finnish net, no conclusion can be drawn about the level of the Gulf of Bothnia. It is connected to the Norwegian coast by a single levelling line, which shows a significant difference in level between the Gulf of Bothnia and the Atlantic. But this evidence is weak, because the line has not been corrected satisfactorily for land uplift and is not controlled by other lines.

The picture presented by the mareographs along the Atlantic coast shows considerable irregularities. Leaving apart the possibilities of each mareograph being affected by systematic influences caused by the shape of the coastline etc.., this irregularity shows that it will be important to get information from more mareographs by connecting them to the net and by examining and testing the results.

The mareographs from the Skagerrak to the English Channel give rather an impression of consistency. If we go further southward, the meaning of the differences between mareographs becomes doubtful because of the poor quality of the net in that region. As an example we may remember that between M 38 and M 39 the difference in g.p.n. corresponds to about 14 cm, and that this difference is hardly significant if we consider the standard deviation of the levelling between them. The result of the poor connection of these mareographs is that if we want to find a mean level for the Atlantic, this level is almost entirely determined by the mareographs between Denmark and Holland.

The Spanish east coast and the French south coast present also an irregular picture, and we see here the same lack of control caused by the weakness of the net. Therefore, the level of the Mediterranean is practically determined by some mareographs that are more strongly connected to the net. It is believed that there should be more evidence before one draws far-reaching conclusions about the difference in level between the Atlantic and the Mediterranean. This evidence could of course be found by means of more precise and more reliable regional levelling and by considering more mareographs on the Iberian peninsula, especially around the straits of Gibraltar.

The following summarising remarks may be made :

a. i. As to the levelling, it would be interesting to study more deeply the assignment of weights to the observations.

ii. There is reason to believe that in some parts of the net more accurate gravity values should be used.

iii. It seems desirable that the composition of the net be studied anew. It does not seem to be essential to use exactly the same lines if a new adjustment is considered, at least as far as computational technique is concerned.

b. It is thought that it is not now possible to arrive at a reliable conclusion about the behaviour of M.S.L. along the Atlantic coast.

c. Without an improvement of the levellings on the Scandinavian peninsula it is not possible to draw a reliable conclusion about a difference in level between the Gulf of Bothnia and the Atlantic.

d. Great care should be taken in drawing conclusions from the observed difference in level between the Atlantic and the Mediterranean.

e. Although the matter belongs to the competence of oceanographers, attention is drawn to the importance of getting a better insight into the definition of M.S.L. and into the accuracy of that definition at the mareographs.

REFERENCES

[1] Notes of the Meeting of Representatives of the Computation Offices, Delft, 28-31 January, 1959. (Stencil, Delft, June 1959).

[2] Baarda, W. Some Bemarks on the Computation and Adjustment of Large Systems of Geodetic Triangulation, Section 1. Bulletin Géodésique, 43, 1957.

[3] Baarda, W., D. de Groot and F. Harkink, Handleiding voor de technische werkzaamheden van het kadaster (Manual for the Technical Activities of the Cadastral Service in the Netherlands), 's-Gravenhage, 1956.

[4] Report on Establishment and Adjustment of European Levelling Networks, presented by Dr. O. Simonsen. (Draft, Copenhagen, September 1959).