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Estimating Canadian vertical datum offsets using GNSS/levelling benchmark information and GOCE global geopotential models

Research article

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Abstract:

The performance of GOCE-based geopotential models is assessed for the estimation of offsets for three regional vertical datums in Canada with respect to a global equipotential surface using the GNSS benchmarks from the first-order vertical control network. Factors that affect the computed value of the local vertical datum offset include the GOCE commission and omission errors, measurement errors, the configuration of the network of GNSS/levelling benchmarks, and systematic levelling errors and distortions propagated through the vertical control network. Among these various factors, the effect of the GOCE omission error on the datum offsets is investigated by extending the models with the high resolution gravity field model *EGM2008* and by means of Canada's official high resolution geoid model *CGG2010*. The effect of the GOCE commission error in combination with errors from the GNSS/levelling data is also examined, in addition to the effect of systematic levelling errors. In Canada, the effect of the GOCE omission error is at the dm-level when computing local vertical datum offsets. The effect of including accuracy information for the GNSS/levelling data and the GOCE geoid heights can be up to 4 cm over the Canadian mainland and at the dm-level for island regions. Lastly, the spatial tilts found in the levelling network can be modelled with a 2-parameter bias corrector model, which reduces the RMS of the adjusted geoid height differences by 4 cm when compared to the RMS of adjusted geoid height differences computed without the use of a bias corrector model. Thus, when computing local vertical datum offsets in Canada, it is imperative to account for GOCE commission and omission errors, ellipsoidal and levelling height errors, as well as the systematic levelling errors of the vertical control network.

Keywords:

Height system unification • CGVD28 • GOCE global geopotential model • GOCE omission error • NAVD88 • vertical datum

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1. Introduction

A vertical datum is the primary component of any vertical reference system for physically meaningful heights; it defines the zero level relative to which the vertical position of a point on the physical surface of the Earth is obtained through geodetic levelling (Vaníček 1991). At present, there are more than one hundred regional vertical datums in the world, many of which are defined by the local mean sea level (MSL) with the assumption that the MSL coincides with the geoid (Balasubramania 1994). The local MSL is determined by averaging long-term sea level records at one or more tide gauge stations connected to the vertical control network. The discrepancy between the geoid and the MSL can reach up to 2 meters (Balasubramania 1994). Hence, vertical datum realization and unification has been amongst one of the main topics of research in geodesy, and has been discussed in detail over the last three decades by Colombo (1980), Rummel and Teunissen (1988), Heck and Rummel (1990), Rapp and Balasubramania (1992), Xu (1992), Rummel and Ilk (1995), Jekeli (2000), and Burša et al. (2004), among others. These studies illustrate that by determining the potential value of a local vertical datum (LVD), it is possible to relate differ-

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ent datums around the world, and also to define a world vertical datum.

However, the realization of datum unification methods has been limited by the accuracy of the required data (Sánchez 2009): orthometric heights, GNSS ellipsoidal heights, local geopotential or geoid heights, gravity anomalies, and oceanographic sea surface models. For example, the relative accuracy of orthometric heights has been 1-2.5 cm/10 km using geometric levelling, 2-2.5 cm/10 km using trigonometric levelling, and 2 cm/10 km using GNSS techniques (Balasubramania 1994). Using static GNSS technologies, ellipsoidal heights typically have a 1-2 cm vertical accuracy. For oceanographic sea surface models, Bingham and Haines (2006) demonstrated that it is possible to generate a model as accurate as 3.2 cm RMS at a spatial scale of 1°. Moreover, the existing classical vertical datums do not support the accuracy requirements of modern geodesy. Their accuracy is about two orders of magnitude (10^{-6} to 10^{-7}) less than that (10^{-9}) of the coordinates of the stations forming the present-day spatial geometrical reference frame (Ihde and Sánchez 2005). The global accuracy of the physical heights is limited by the definition of the LVDs that refer to local isolated MSL determined at different time epochs.

Gravity field information is also subject to errors, some of them due to errors and LVD biases in physical heights. Free-air gravity anomalies contain systematic errors from different sources: biases generated by gravity datum inconsistencies, vertical and horizontal datum inconsistencies, and distortions from a simplified free-air reduction. The contribution of each source can reach 0.1-0.2 mGal. These systematic errors affect the medium gravity spectrum and propagate into the spectrum of the geoid between degrees 10 and 200 (Heck 1990). Additionally, satellite gravity missions, such as the Challenging Mini-Satellite Payload (CHAMP) and the Gravity Recovery and Climate Experiment (GRACE), have provided static gravity field solutions at the dm-level geoid accuracy up to degree and order 60 and 120, respectively (Sneeuw and Schaub 2004). The goal of the European Space Agency's (ESA) Height System Unification (HSU) project is to unify and re-define existing vertical datums in relation to only one equipotential surface: the Gravity field and steady-state Ocean Circulation Explorer (GOCE) geoid. The GOCE satellite was launched on March 17, 2009, in order to measure the Earth's gravity field at an unprecedented accuracy and spatial resolution (Drinkwater et al. 2003). Specifically, the GOCE mission aims to determine the gravity anomalies with an accuracy of 1 mGal and the geoid with an accuracy of 1-2 cm at a spatial resolution better than 100 km (`www.esa.int/esaLP/ESAYEK1VMOC_LPgoce_0.html`).

Thus, within the context of using new high-accuracy satellite-borne gravity missions for the purpose of vertical datum unification, the potential and the LVD offsets for three vertical datums in Canada, i.e., CGVD28, NAVD88, and Nov07, are computed with respect to a global equipotential surface using the GOCE geoid. In this respect, we assess the effect of the GOCE omission errors, the effect of the GOCE commission error in combination with the errors of the GNSS/levelling data, as well as the effect of the systematic levelling errors on the LVD offsets. To evaluate the effect of the GOCE omission error on the computed LVD offset, the offset value obtained by means of the GOCE geoid is compared with the offset value obtained with the high resolution gravitational model *EGM2008* (Pavlis et al. 2012) and the Canadian Gravimetric Geoid of 2010 (*CGG2010*, M. Véronneau and J. Huang, presented at the CGU Annual Scientific Meeting, Banff, Alberta, Canada, May 15-18 2011). Section 2 of the paper reviews the methodology and data utilized. Results are presented and discussed in Section 3. A summary of the main outcomes of this study and conclusions are given in Section 4.

2. Experimental procedures

Different approaches can be used for the unification of the local vertical datums. They can be connected by means of precise geodetic levelling combined with gravity measurements along the levelling lines. An example of this approach is the United European Levelling Network, (EUVN, Ihde et al. 2000), which consists of the national levelling networks of 27 European countries. Ocean levelling using oceanographic information (e.g., Rummel and Ilk 1995, Thompson et al. 2009) is another approach that can be used to connect LVDs without connection points on land. The Geodetic Boundary Value Problem (GBVP) solution is the rigorous method for LVD unification (Rummel and Teunissen 1988, Gerlach and Rummel 2012). GNSS-determined ellipsoidal heights, levelling heights from precise geodetic levelling, and geoid heights from local gravity data and global geopotential models (GGMs) are used in the GBVP method to unify the LVDs. The maximum degree of the GOCE GGM can define a residual Stokes's kernel used for integration of the local biased gravity anomalies. In this case, knowledge of gravity anomalies from different LVD zones is not required as the error of omitting the so-called "indirect bias term" stays below the 1 cm level (Gerlach and Rummel 2012). Our investigations also show that the omission of the indirect bias term in the computation of the North American datum offsets leads to less than 1 cm offset error for a GGM of a maximum spherical harmonic degree 70 or higher. Hence the geoid heights in our study are obtained directly from the GOCE GGMs without taking into account the bias of the gravity anomalies from different LVD zones. The approach followed herein is similar to the studies by Burša et al. (2001, 2004) and Kotsakis et al. (2011), among others.

2.1. Methodology

In this study the offset of a LVD j, δN_P^j, is computed with respect to a global equipotential surface, which is defined by a global conventional potential value W_0, using

$$\delta N_P^j = -N_0 + (h_P - H_P^j - N_P), \tag{1}$$

where h_P is the ellipsoidal height, H_P^j is the orthometric height of point P with respect to the LVD, and N_P is the geoid height computed from a GOCE GGM, GOCE GGM plus local gravity and topographic information, the *EGM2008* model, or interpolated from a regional gravimetric geoid model. N_0 is the zero-order term of the geoid height (Heiskanen and Moritz 1967, p. 100) used for rescaling the geoid height as follows:

$$N_0 = \frac{\delta GM}{R\gamma} - \frac{\Delta W_0}{\gamma}, \tag{2}$$

which is a constant that depends on δGM, the difference in the geocentric gravitational constant GM and the GM^e of the normal ellipsoid, and ΔW_0, the difference between the potential W_0 of the reference equipotential surface and the potential of the normal ellipsoid U_0.

The geopotential difference $\delta W_P^j = W_0^j - W_0$ is then computed by multiplying the computed offset by the normal gravity on the ellipsoid γ:

$$\delta W_P^j = \gamma \delta N_P^j. \tag{3}$$

W_0^j is the potential of the local datum, which is determined by adding the geopotential difference δW_P^j to the potential W_0 of the reference global equipotential surface. If the observations are equally-weighted, the LVD offset is computed by averaging the right hand side of Eq. (1) over all data points.

Preferably, one would obtain the geoid height N_P from a high-accuracy satellite-only GGM, such as the GOCE models, and evaluate the medium- to short-wavelength geoid signals using local gravity anomalies and topographic information in a remove-restore computational scheme. The Canadian geoid model *CGG2010* is an example of an optimal combination of a satellite-only GGM, local gravity, and topographic information. The *CGG2010* geoid heights are available on a 2′ by 2′ grid for a region that covers 20°N to 80°N and -170°W to -10°W. Hence, *CGG2010* can be utilized to assess the magnitude of the omission error of the GOCE GGM in a large part of North America.

One can also use the *EGM2008* model to approximate the omission error of the GOCE models (Gruber et al. 2011). However, it should be noted that *EGM2008* (maximum degree and order 2190) is not able to model a gravity field signal with half-wavelengths smaller than 9 km. When the geoid is computed from *EGM2008*, the global average omission error is approximately 4 cm although it can be larger in mountainous terrain (Jekeli 2009). In other words, the effect of the omission error in low-lying terrain tends to be smaller when compared to mountainous areas (see, e.g., Hirt et al. 2010) due to the fact that the topography of the Earth is the main source of high-frequency gravity field signals (Forsberg 1984). The residual terrain model (RTM) technique (Forsberg and Tscherning 1981, Forsberg 1984, 1985) can be used to model the high-frequency signals not provided by the *EGM2008* model. In regions with sufficient terrestrial gravity data coverage, such as Canada or the USA, the remove-restore method allows for a more accurate modelling of the high frequency components of the gravity field. Thus, *CGG2010* is a better candidate than *EGM2008* to quantify the contribution of the high frequencies of the gravity field that are not modelled by the GOCE GGMs.

We should emphasize that *CGG2010* and *EGM2008* are not completely independent from each other. *EGM2008* uses the high-quality Canadian local gravity information that is used in the computation of *CGG2010*. On the other hand, *CGG2010* utilizes *goco01s* (Pail et al. 2010) and *EGM2008* from degree 2 to 224 using a weighted averaging based on the coefficient standard deviations and *EGM2008* up to degree and order 2190 (Huang J., Natural Resources Canada, personal communication, Nov. 29, 2012). Therefore, one can expect that the effect of the GOCE omitted signal on the LVD offsets will not differ significantly when it is evaluated by means of *EGM2008* and *CGG2010*.

2.2. Errors affecting LVD offset accuracy

The LVD offsets computed at the GNSS/levelling benchmarks of the levelling network in the datum zone j (see Eq. 1) will contain systematic and random measurement errors, as well as errors from the gravity field models. A biased LVD offset (i.e., an offset value that differs from the true LVD offset value) may result in the presence of systematic effects and spatially correlated errors in the height data, in which case $h_P - H_P^j - N_P$ will not follow the typical trend of a constant offset, but may instead reveal spatial tilts or even a more complex oscillatory pattern over the network of GNSS/levelling benchmarks (Kotsakis et al. 2011). Systematic errors that can contribute to this problem include datum inconsistencies between the ellipsoidal heights and the geoid heights resulting from the use of different reference ellipsoids in the geoid model and ellipsoidal heights, geometrical distortions in the levelling height data due to over-constraining the LVD to several tide gauge stations (e.g., CGVD28 in Canada), long and medium wavelength errors in the geoid model, accumulated systematic errors in the levelling network (Entin 1959), improper modelling of temporal height variations, and the inconsistent treatment of permanent-tide in the geoid, physical, and ellipsoidal heights (Ekman 1989, Mäkinen and Ihde 2009). Other factors include the uncertainty of the Earth's geocentric constant, which stems from the uncertainty of the zero-degree term of the Earth's gravity potential, corresponding to a vertical uncertainty of more than 1 cm for the zero-height surface of a vertical datum (Kotsakis et al. 2011). Additionally, commission errors of the GGMs for wavelengths that exceed the size of the test area will act as a bias on the LVD offset estimate and cannot be reduced by increasing the sampling distribution of the GNSS/levelling benchmarks. This may be especially problematic for small test regions, such as the independent levelling networks of Vancouver Island and Newfoundland (see Fig. 2).

The removal of the systematic effects in the height data can be performed through appropriate corrections and spatial de-trending of the raw geoid height differences or simultaneously with the LVD offset using the extended observation equation (Kotsakis et al. 2011):

$$\delta N_P^j + a_P^T x + v_P = -N_0 + (h_P - H_P^j - N_P), \qquad (4)$$

where the term $a_P^T x$ absorbs the systematic errors through a set of parameters $\boldsymbol{x}$ and $\boldsymbol{a}_P$ is a vector of known coefficients dependent on the spatial position of the GNSS/levelling benchmarks; v_P represents the random error of the geoid height difference $h_P - H_P^j - N_P$ at point P. For a comprehensive discussion regarding examples of parametric models used for the description of systematic effects one may refer to Fotopoulos (2003).

The inseparability between the δN_P^j term and the bias parameter of the corrector model remains a problem for the practical determination of local datum potentials and offsets, and as such the estimation/correction of systematic effects may be more effective when performed before the computation of the LVD offset.

2.3. Data

The main datasets include physical heights at benchmarks obtained from levelling and gravimetric measurements, ellipsoidal heights obtained from GNSS on the benchmarks, and geoid heights obtained from GGMs or from regional gravimetric geoid models (see Table 1).

For the remaining sections of the paper, the third generation time-wise model *go_cons_gcf_2_tim_r3* is abbreviated as *tim_r3*.

2.3.1. Vertical datums in Canada

Nov07 is a vertical height reference frame for Canada, realized for the purpose of validating geoid models in North America. The heights of benchmarks are Helmert orthometric, computed by scaling the geopotential numbers with the Helmert approximation of the mean gravity along the plumb line (Heiskanen and Moritz 1967, p.167). It should be noted that Nov07 is not an official vertical datum, and is the latest realization of a series of minimum-constraint adjustments of the federal first-order levelling network. The main network covers the continental main land (CML), and includes independent networks for Newfoundland (NFD), Vancouver Island (VAN), and various other islands. In addition, there is a series of independent local networks around tide gauges in the northern region of Canada. The fixed station for the CML network is the former tide gauge in Pointe-au-Père, Quebec (QC), on the lower St. Lawrence River. Similarly, each of the other regions (i.e., NFD, VAN, etc.) has their adjustments tied to their own respective fixed tide gauge stations. It was found that the best approach to decrease the systematic error for Nov07 was to adjust together only the most recent levelling measurements that allow a continuous network between Vancouver and Halifax, resulting in a discrepancy between the two coasts of 80 cm, which represents about 20 to 30 cm of systematic error over approximately 6000 km of levelling lines (Véronneau 2012). The remaining 50-60 cm accounts for the separation between the mean water levels on the west and east coasts of Canada as indicated by gravimetric geoid models that incorporate both Gravity Recovery and Climate Experiment (GRACE) and GOCE data (Véronneau 2012). This separation was first reported by Sturges (1967), where it was shown that MSL values at tide gauges on the Pacific coast appeared to be systematically 60-70 cm higher than those of similar latitude on the Atlantic coast, which was later shown to be caused by a combination of ocean density differences and boundary current effects (Sturges 1974).

The Canadian Geodetic Vertical Datum of 1928 (CGVD28) is the official vertical datum of Canada, which has been realized through levelling and is accessible through approximately 80,000 benchmarks mostly distributed in southern Canada. It is based on an adjustment of levelling measurements prior to 1928 with constraints to the mean sea level at six tide gauges: Vancouver (BC), Prince-Rupert (BC), Point-au-Père (QC), Halifax (NS), Yarmouth (NS), and New York City (Véronneau 2006). Since the original adjustment, all levelling measurements consisting of re-observations or extensions have been processed according to the same procedure and constrained as the 1928 original adjustment. The CGVD28 heights are computed using normal gravity values based on latitude instead of actual gravity measurements, hence the heights are said to be “normal-orthometric heights” though they are neither orthometric nor normal heights (Bomford 1971, p.228), and as such CGVD28 does not coincide with the geoid or the quasi-geoid (Véronneau 2006). Moreover, the realization of the CGVD28 does not take into account the sea surface topography at the tide gauge stations, the fact that mean sea level is rising due to melting of glaciers and thermal expansion, surface expression of and ruptures due to earthquakes, frost heave, local instabilities, and the fact that land elevation is changing due to the rebound/subsidence of the Earth’s crust (i.e., post-glacial rebound). Additionally, the levelling data used in CGVD28 are not corrected for systematic errors due to atmospheric refraction, rod calibration, rod temperature, and the effects of solar and lunar tides on the Earth’s geopotential surfaces. Due to various correction omissions, approximations, and the fact that the vertical control network was established over time in a piece-wise manner, the CGVD28 datum has a national distortion that ranges from -65 cm in Eastern Canada to 35 cm in Western Canada with respect to an equipotential surface (Véronneau and Héroux 2006). Currently, the network is characterized by a rapid rate of degradation due to destruction and loss of physical markers and limited maintenance as Canada is planning to implement a geoid-based GNSS-accessible vertical datum by 2013 (Véronneau et al. 2006).

The North American Vertical Datum of 1988 (NAVD88) is the vertical datum established for vertical control surveying in the USA (NGS 1996). It was the result of a joint effort in the 1970s and 1980s by the governmental agencies of USA, Canada, and Mexico to unify the vertical control networks on the continent. The NAVD88 was established by the minimum-constraint adjustment of geodetic levelling observations in Canada, USA, and Mexico. It held fixed the height of the primary tidal benchmark at Rimouski,

Table 1. Global geopotential models, regional geoid models, and their supporting background models utilized for the evaluation of LVD offsets.

Model	Maximum D/O	Data Source	References
go_cons_gcf_2_tim_r3	250	GOCE time-wise approach based on 12 months of data	Pail et al. 2011
goco03s	250	based on 12 months of GOCE data, 7 years of GRACE data, 8 years of CHAMP data and 5 years of SLR	Mayer-Gürr et al. 2012
goco01s	224	based on 2 months of GOCE data and 7 years of GRACE data	Pail et al. 2010
itg-grace2010s	180	based on 7 years of GRACE data	Mayer-Gürr et al. 2010
itg-grace03s	180	based GRACE data collected from September 2002 to April 2007	Mayer-Gürr et al. 2007
EGM2008	2190	based on itg-grace03s global geopotential model, surface gravimetry, and altimetry	Pavlis et al. 2012
CGG2010	Gravimetric	based on terrestrial gravity data and global models goco01s and EGM2008	Véronneau and Huang 2011

Quebec, Canada. Additional tidal benchmark elevations were not used due to the demonstrated spatial (and temporal) variations in the sea surface topography. NAVD88 heights are Helmert orthometric heights. The NAVD88 datum was never officially adopted in Canada due its large east-west tilt of 1.5 m from the Atlantic to Pacific coasts, possibly due to the accumulation of systematic errors in the levelling network (Véronneau and Héroux 2006).

2.3.2. GNSS benchmark distribution

For this study, two subsets of GNSS/levelling benchmarks are used. The first dataset refers to 308 benchmarks (BMs), where the original ellipsoidal heights and coordinates were given in ITRF2005 (Altamimi et al. 2007) epoch 2006.0 and come from the newest adjustment of the GPS SuperNet network in Canada (Craymer and Lapelle 1997). These have been updated to ITRF2008 epoch 2008.0, with the effect of post-glacial rebound taken into account using the velocities from the Argus and Peltier (2010) *GEODVEL1b GPS* solution. These benchmarks have physical heights obtained from levelling that refer to CGVD28, NAVD88, and the Nov07 vertical datums. The distributions of the benchmarks are shown in Fig. 1.

The next dataset refers to levelling data given with respect to the Nov07 vertical datum. There are three main regions: Canadian Mainland (CML), Newfoundland (NFD), and Vancouver Island (VAN). Figure 2 shows the distributions of these benchmarks.

The Canadian mainland (CML) consists of 1315 benchmarks, while the Vancouver Island (VAN) network has 26 benchmarks, and the Newfoundland (NFD) network contains 34 benchmarks. The topography of Vancouver Island can be considered medium rugged terrain, where coastal elevations range from 100 to 500 m while the average elevation of the interior of the island ranges from 500 to 1500 m, reaching a maximum of 2000 m (Natural Resources Canada 2007). In contrast, the terrain of Newfoundland is much more flat, ranging mostly from 100 to 500 m, reaching a maximum of 700 m in a few areas (Natural Resources Canada 2007). For the Canadian mainland, the relief of the terrain ranges from mean sea level to 500 m for eastern and most of central Canada, while the topography becomes quite rugged for the western portion of the country (i.e., parts of Alberta, British Columbia, and Yukon) where the elevation ranges from 500 m to 3000 m, reaching a maximum of 5000 m or more in a few selected areas (Natural Resources Canada 2007).

3. Results and discussion

GOCE model evaluation studies (e.g., Hirt et al. 2012, Ince et al. 2012) have shown that GOCE GGMs perform well only up to degree and order 180. Ince et al. (2012) have shown that the third generation GOCE models have the best agreement in Canada when compared with geoid undulations obtained from GNSS and levelling. For this study, the third generation GOCE models *goco03s* and *tim_r3* (see Table 1) have been used up to degree and order 180. The *tim_r3* model does not utilize a background model, and can be considered to be a 'pure' GOCE model (i.e., only contributions from the GOCE satellite) while the *goco03s* can be considered a satellite-based combined model as it utilizes observations from several different complementary sources (see Table 1). Additionally, a GRACE-based GGM (i.e., *itg-grace2010s*) with maximum degree and order 180 has also been used in order to evaluate LVD offsets so that one may compare the performance of a GOCE only based model and GRACE only based model. Pail et al. (2010) have shown that GRACE is the most important dataset for the modeling of the long wavelength components of the gravity field (i.e., degree and order 2 to 100) whereas GOCE is a significant contributor from degree and order 100, and is even more effective beyond degree and order 150 in *goco01s*. Due to the improved performance in the medium wavelength range (i.e., degree and order 100 to 200), utilizing a combined GRACE and GOCE GGM for LVD offset computations is expected to yield a more accurate geoid signal in this particular spectral range when compared to *EGM2008*.

In order to assess the performance of the GOCE-based models in evaluating the LVD offsets for the Canadian vertical datums, the results are compared with those obtained with high resolution regional geoid models, in particular *CGG2010* (see Section 2.1). Moreover, the GOCE models are extended from degree and order 181 to 2190 using *EGM2008* in order to evaluate the effect of

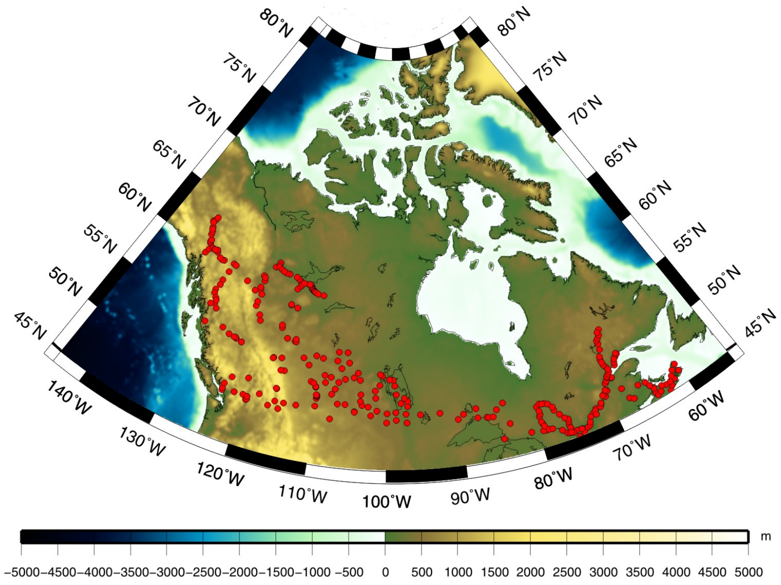

Figure 1. Distribution of 308 GNSS/levelling benchmarks common in Nov07, CGVD28, and NAVD88 vertical datums.

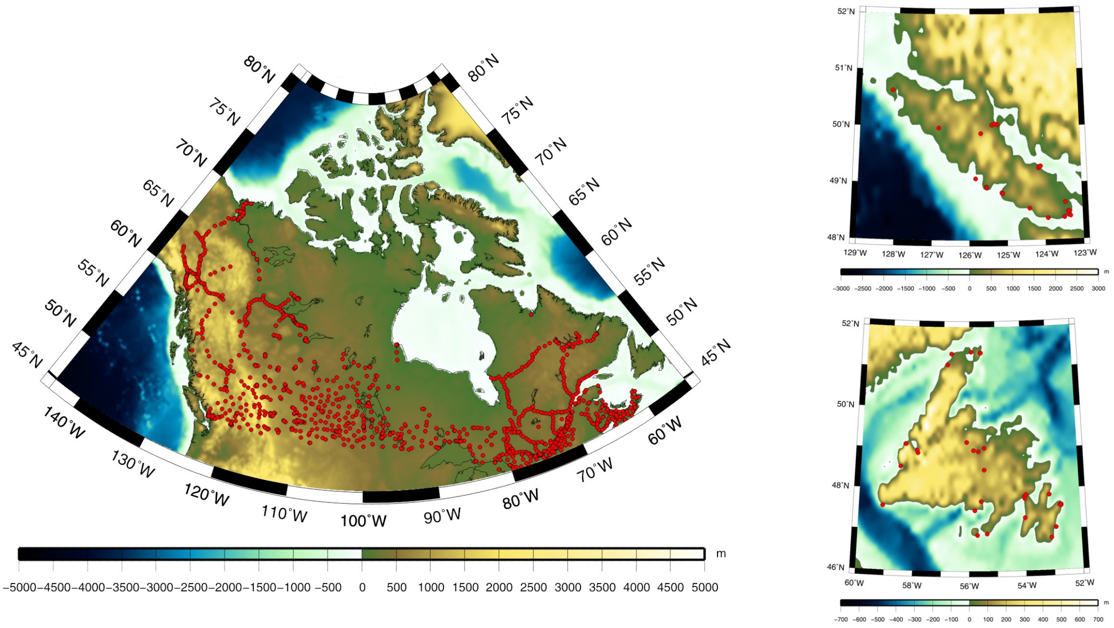

Figure 2. Distribution of Nov07 Benchmarks (**Left:** CML **Top Right:** VAN **Bottom Right:** NFD).

the omission error on the computed LVD offsets. All computations have been implemented with the GRS80 ellipsoid parameters. The 2010 International Earth Rotation and Reference Systems Service's (IERS) conventional value of 62636856.00 m^2/s^2 (Petit and Luzum 2010) was used for W_0 defining the global equipotential surface. The height data used in this study are all in a tide-free system.

3.1. Nov07, CGVD28, and NAVD88 LVD offsets evaluated with 308 GNSS/levelling benchmarks

Table 2 presents the LVD offsets of Nov07, NAVD88, and CGVD28 for the first data set of 308 GNSS/levelling benchmarks distributed mainly along the southern region of Canada (see Fig. 1). The results are shown in terms of the potential difference between the reference conventional surface and the local datum (i.e., δW_P^j) and the offset of the LVD (i.e., δN_P^j) with their respective standard deviations in both Tables 2 and 3. For the least-squares evaluation of Eq. (1), a unit weight matrix has been used thus assuming a constant and uncorrelated noise in the known heights at all data points. The error estimates shown in the tables indicate only a formal statistical accuracy.

From Table 2, it is evident that when the GOCE models are only expanded up to degree and order 180, the effect of the omission error is quite significant in Canada. For example, the Nov07 offset computed with $goco03s{+}EGM2008$ is -44.8 cm and the offset computed with $tim\_r3{+}EGM2008$ is -45.1 cm while the GOCE only models yield -58.2 cm and -58.6 cm, respectively. Thus, the effect of the GOCE GGM omission error on the offsets can be quantified as 13 cm between the GOCE models and the GOCE+$EGM2008$ models for the distribution of these 308 GNSS-levelling data points. Both $goco03s$ and tim_r3 perform very similarly as shown in Table 2; the difference between the evaluated LVD offsets is less than 1 cm for all three vertical datums. The similar performance is expected as both models use the same 12 months of GOCE data and similar data pre-processing (e.g., same de-aliasing models and GNSS orbits; Pail et al. 2011). Similarly to GOCE, the $itg\text{-}grace2010s$ omission error affects the computed offsets by 13 cm.

It is expected that the results obtained using the GOCE+$EGM2008$ and GRACE+$EGM2008$ models will be similar to the results obtained using the $EGM2008$ model, as their high frequency contributions are a result of the $EGM2008$ spherical harmonic coefficients from degree and order 181 to 2190. Likewise, it is expected that the results obtained with $CGG2010$, $EGM2008$, and the GOCE+$EGM2008$ models will be similar. For example, the Nov07 offset evaluated with $EGM2008$ is -44.8 cm and with $CGG2010$ it is -45.2 cm. Similar to $EGM2008$, $CGG2010$ yields a 13 cm difference between the offsets computed with $goco03s$ and tim_r3, but 15 cm when $itg\text{-}grace2010s$ is used. Therefore, based on the smaller offset difference, it can be concluded that $goco03s$ or tim_r3 should be preferred to $itg\text{-}grace2010s$ for the LVD offset computations.

The higher frequency information in the GOCE+$EGM2008$ models also reduces the formal standard deviation of the estimated LVD offsets. For example, the standard deviation of the Nov07 offset using $goco03s$ is 2.1 cm while it reduces to 0.7 cm in the $goco03s{+}EGM2008$ case. This is because the geoid heights computed with the GOCE+$EGM2008$ models are more consistent with the GNSS-levelling geoid heights than the geoid heights computed with the truncated GOCE models of degree 180. Moreover, it can also be observed that the standard deviations of the estimated NAVD88 (2.2 cm) and CGVD28 (1.7 cm) offsets are larger than the standard deviation of the Nov07 offset (0.7 cm) for the $tim\_r3{+}EGM2008$ case. This can be explained by the existing systematic errors and local datum distortions in NAVD88 and CGVD28 described in Section 2.3.1.

3.2. Nov07 LVD offsets evaluated for the Canadian mainland, Newfoundland, and Vancouver Island regions

Table 3 compares the Nov07 offset computed with two high resolution models (i.e., $CGG2010$ and $goco03s{+}EGM2008$), as well as the $goco03s$ model truncated at degree and order 180. The Canadian mainland (CML) dataset contains 1315 benchmarks, compared to the 308 utilized for the results obtained in Table 2. Note that for the sub-regions of VAN and NFD, the offset is evaluated for a local level surface that is different from that of the Nov07 CML network.

Firstly, it can be seen that the effect of the omission error actually increases when utilizing the CML dataset compared to the data set of 308 points. For example, the offset difference when using the $goco03s$ only and the $goco03s{+}EGM2008$ model is 14.7 cm; a very similar difference of 14.2 cm is computed when $goco03s$ is compared against $CGG2010$. This is an increase of more than 1 cm for the respective cases in the 308 data points study. This is likely due to the fact that a portion of the newly added points are located in the rugged terrain of western Canada. However, this difference cannot be considered statistically significant. This result once again shows that the addition of the higher frequencies of the gravity field signal to the GOCE GGMs and the configuration of the GNSS-levelling network are important factors for the accurate determination of the LVD offsets in Canada. Thus, when using GOCE-based GGMs for the evaluation of LVD offsets in Canada, it is recommended that a rigorously combined GOCE GGM and local gravity and terrain data should be used.

It can be seen that increasing the number of GNSS/levelling benchmarks when using high frequency gravity information has little effect on the LVD offsets computed with CML GNSS/levelling benchmarks. For example, the Nov07 offset evaluated with 308 GNSS/levelling is -45.2±0.6 cm while the Nov07 offset evaluated with 1315 GNSS/levelling is -45.0±0.3 cm when using $CGG2010$. The difference is only 0.2 cm and is not significant. Likewise, for the Nov07 offset evaluated with $goco03s{+}EGM2008$ and 308 GNSS/levelling benchmarks and those evaluated with the 1315

Table 2. Potential and vertical datum offsets for Nov07, NAVD88, and CGVD28 vertical datum evaluated with 308 GNSS/levelling benchmarks.

Vertical Datum	δW^j (m^2/s^2)	δN^j (cm)	δW^j (m^2/s^2)	δN^j (cm)
	goco03s nmax:180		goco03s+EGM2008 nmax:180 + 181 to 2190	
Nov07	-5.71± 0.20	-58.2 ± 2.1	-4.40 ± 0.06	-44.8 ± 0.7
NAVD88	-9.23 ± 0.30	-94.0 ± 3.1	-7.91 ± 0.22	-80.6 ± 2.2
CGVD28	-2.77 ± 0.20	-28.3 ± 2.0	-1.46 ± 0.16	-14.9 ± 1.7
	tim_r3 n_{max}: 180		tim_r3+ EGM2008 n_{max}:180 + 181 to 2190	
Nov07	-5.75 ± 0.20	-58.6 ± 2.1	-4.43 ± 0.06	-45.1 ± 0.7
NAVD88	-9.26 ± 0.30	-94.4 ± 3.1	-7.94 ± 0.22	-81.0 ± 2.2
CGVD28	-2.81 ± 0.21	-28.6 ± 2.1	-1.49 ± 0.16	-15.2 ± 1.7
	itg-grace2010s n_{max}: 180		itg-grace2010s + EGM2008 n_{max}: 180 + 181 to 2190	
Nov07	-5.88 ± 0.23	-59.9 ± 2.3	-4.57 ± 0.12	-46.5 ± 1.3
NAVD88	-9.39 ± 0.32	-95.7 ± 3.3	-8.08 ± 0.24	-82.3 ± 2.5
CGVD28	-2.94 ± 0.22	-30.0 ± 2.3	-1.63 ± 0.19	-16.6 ± 1.9
			CGG2010	
Nov07	–	–	-4.42 ± 0.06	-45.2 ± 0.6
NAVD88	–	–	-7.92 ± 0.21	-81.0 ± 2.1
CGVD28	–	–	-1.49 ± 0.17	-15.3 ± 1.7
			EGM2008 n_{max}:2190	
Nov07	–	–	-4.39 ± 0.06	-44.8 ± 0.6
NAVD88	–	–	-7.91 ± 0.21	-80.6 ± 2.2
CGVD28	–	–	-1.45 ± 0.16	-14.8 ± 1.7

Table 3. Potential and vertical datum offsets and their accuracy for Nov07 vertical datum evaluated with GNSS/levelling benchmarks from CML, NFD, and VAN regions of Canada.

Regions	δW^j (m^2/s^2)	δN^j(cm)
	CGG2010	
CML (1315)	-4.40 ± 0.03	-45.0 ± 0.3
NFD (34)	-4.31 ± 0.19	-44.1 ± 1.9
VAN (26)	-3.81 ± 0.13	-38.9 ± 1.3
	goco03s n_{max}: 180	
CML (1315)	-5.81 ± 0.11	-59.2 ± 1.1
NFD (34)	-4.29 ± 0.49	-43.7 ± 5.0
VAN (26)	-1.02 ± 1.45	-10.4 ± 14.8
	goco03s+EGM2008 n_{max}: 180 + 181 to 2190	
CML (1315)	-4.36 ± 0.04	-44.5 ± 0.4
NFD (34)	-3.28 ± 0.10	-33.5 ± 1.1
VAN (26)	-4.05 ± 0.11	-41.3 ± 1.1

GNSS/levelling benchmarks the difference is only 0.3 cm. The greatest difference is seen in the *goco03s* case only where the difference between the estimated offsets for the two different networks is 1 cm, with a 1 cm decrease in the standard deviation for the network consisting of 1315 GNSS/levelling.

However, the effects of the limited geographical coverage and the sparse and irregular distribution of the GNSS/levelling benchmarks are seen in the offsets estimated on the two islands (i.e., VAN and NFD). For example, the offset evaluated for VAN using *goco03s* only is 10.4±14.8 cm when compared to -38.9±1.3 cm with *CGG2010* or -41.3±1.1 cm with *goco03s+EGM2008*. Using the 3-sigma test, it can be seen that the offset computed with *goco03s* yields a statistically insignificant LVD offset for Vancouver Island as the estimated LVD offset is smaller than three times its error. This can be further explained by the effect of the GOCE omission error on the LVD offset of Vancouver Island, which would be quite significant due to its rugged terrain described in section 2.3.2. For Newfoundland, the offset computed with *goco03s* is -43.7±5.0 cm while *CGG2010* yields a height offset of -44.1±1.9 cm and the *goco03s+EGM2008* model yields -33.5±1.1 cm. The approximate 10 cm difference between the high resolution models (*goco03s+EGM2008* and *CGG2010*) can be explained by the fact that there is approximately a 10 cm difference in the geoid heights of *CGG2010* and *EGM2008* for the NFD GNSS/levelling benchmarks. The cause of this difference requires further investigation in a future study. Thus, for each of the islands, the computed potential and offset for the Nov07 datum illustrate that the GGM inaccuracies and the measurement errors cannot average out over the limited geographic coverage and number of GNSS/levelling benchmarks found in each region.

Lastly, Table 4 shows the LVD offsets computed with the inclusion of the ellipsoidal height, orthometric height, and geoid height error estimates using *goco03s* to degree and order 180 in order to evaluate the effect of including accuracy information on the LVD offset estimations. When propagating the commission errors from the GOCE model to the geoid heights only the variances of the spherical harmonic coefficients were used.

Table 4. LVD offset for Nov07 datum for CML, NFD, and VAN regions using *goco03s* (n_{max}: 180) and error information for the ellipsoidal, orthometric, and geoid heights.

Region	δN^j (cm) without error information	δN^j (cm) with error information
CML	-59.2 ± 1.1	-63.2 ± 1.0
NFD	-43.7 ± 5.0	-56.9 ± 2.5
VAN	-10.4 ± 14.8	77.3 ± 8.9

Based on the results presented in Table 4, it can be seen that using the GOCE commission error information up to degree and order 180, in combination with the error estimates for the ellipsoidal and levelling heights, results in a 4 cm difference in the LVD offset with respect to the LVD offset estimated without any error information for the CML region. The effect is more pronounced for Newfoundland, where the difference is 13 cm. One of the reasons contributing to this difference can be explained by the fact that the geographical coverage of the NFD network is much smaller when compared to the geographic coverage of the CML network, and therefore it may be affected by the commission errors of the GGM wavelengths that exceed the size of the test area. The large difference of almost 88 cm for Vancouver Island may be due to the uncertain reliability of the control data and their errors for the VAN network.. Additionally, using only the variances of the spherical harmonic coefficients may also have an effect as the inclusion of a full variance co-variance matrix may yield different results. Both of these factors require further investigations.

3.3. The effect of systematic errors on Nov07 LVD offsets evaluated for the Canadian mainland

Lastly, we examine the systematic effects in the height data, and their effect on the estimated LVD offset. The original geoid height differences using Nov07 orthometric heights and the *EGM2008* geoid heights are plotted in Fig. 3 in order to show the systematic spatial tilt present in the GNSS/levelling network. Figure 3 shows that there is a strong east-west tilt whereas the north-south tilt appears to be less significant. In order to see this more clearly, S-N and W-E profiles of these spatial tilts are shown in Fig. 4.

For the evaluation of the LVD offset, Eq. (4) is used with Eq. (5), (6), (7), and (8) for the bias corrector term (Kotsakis et al. 2011):

Null model:

$$a_P^T x = 0 \tag{5}$$

When using Eq. (5) no systematic errors or other biases are modeled within the height data. This is what has been applied for the results presented in Section 3.1 and 3.2.

1-Parameter Model:

$$a_P^T x = \delta s H_P^j \tag{6}$$

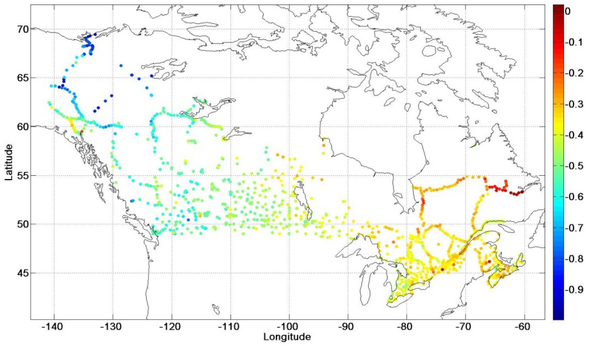

Figure 3. Geoid height differences $h\text{-}H\text{-}N^{EGM2008}$ (in meters) for the CML Nov07 GNSS/levelling benchmarks.

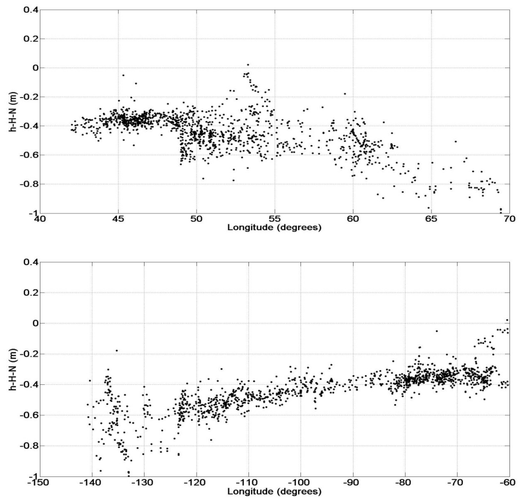

Figure 4. Systematic spatial tilts in S-N direction (top) and W-E direction (bottom) in the original geoid height differences $h\text{-}H\text{-}N^{EGM2008}$, unit meter.

Kotsakis et al. (2011) warns that the application of this systematic model will only be successful in a test network that has a significant height variability so that the offset parameter δN_P^j and the scale parameter δs can be sufficiently separated through the least-squares estimation of Eq. (4).

2-Parameter Model:

$$a_P^T x = x_1(\phi_P - \phi_0) + x_2(\lambda_P - \lambda_0)\cos\phi_P \tag{7}$$

For the model in Eq. (7), ϕ_P is the latitude of the GNSS/levelling benchmark and λ_P is the longitude; the overall tilt consists of a N-S component (i.e., parameter x_1) and a W-E component (i.e., parameter x_2) with respect to the centroid of the test network (ϕ_0, λ_0).

Table 5. Estimated LVD offset (cm) with various bias corrector models and their respective RMS of adjusted geoid height differences (cm).

Model	Null	*RMS*	1-Parameter	*RMS*	2-Parameter	*RMS*	Combined	*RMS*
CGG2010	-45.0 ± 0.3	*12.2*	-41.1 ± 0.5	*11.8*	-45.0 ± 0.2	*8.3*	-47.5 ± 0.4	*8.1*
goco03s+EGM2008	-44.5 ± 0.4	*13.6*	-39.4 ± 0.5	*12.8*	-44.5 ± 0.2	*8.1*	-47.3 ± 0.4	*7.9*

Combined Model:

$$a_P^T x = x_1(\phi_P - \phi_0) + x_2(\lambda_P - \lambda_0)\cos\phi_P + \delta s H_P^j \qquad (8)$$

The combined model consists of a combination of the 1-parameter and the 2-parameter model.

The LVD offsets and the root-mean-square (RMS) error values, computed using the adjusted geoid height differences to show the consistency among the ellipsoidal, orthometric, and geoid heights, are presented in Table 5 for each of the four bias corrector term models.

The CML test network does not have significant height variability throughout the entire network; this is especially the case for the eastern and central parts of the country. Therefore, one does not expect to see a significant improvement in the consistency among the ellipsoidal, orthometric, and geoid heights when using the 1-parameter model. This is shown in Table 4, where the change in the RMS between the null and 1-parameter model can hardly be considered significant. The use of the two parameter model shows the most significant improvement when compared to the use of the null or even the 1-parameter model. Again, the addition of the scale parameter in the combined model shows no significant improvement over the 2-parameter model. Therefore, for the CML network, the 2-parameter model can be considered sufficient for modelling the spatial tilts found in the network. The N-S tilt and the W-E tilt (w.r.t to the centroid of the network) evaluated with the 2-parameter model using *CGG2010* is -0.26 cm/degree and 0.52 cm/degree, respectively. Likewise, the components are -0.14 cm/degree and 0.69 cm/degree when using the *goco03s+EGM2008* GGM.

4. Conclusions

The objective of the paper was to study the performance of the most recent GOCE-based GGMs when computing the potential and height datum offset of three regional vertical datums evaluated with respect to a global equipotential surface. In order to accomplish this, the potential and geoid height differences were evaluated with a GOCE only satellite model (i.e., *tim_r3*), a GOCE satellite only combined model (i.e., *goco03s*), a GRACE only model (i.e., *itg-grace2010s*), the high resolution gravitational model *EGM2008*, and the high resolution regional gravimetric geoid model *CGG2010*.

It can be concluded that the effect of the truncation degree of the GOCE model is significant in Canada—the effect of the omission error is at the decimeter level. This result indicates that the contributions of the higher frequencies of the gravity field is very important when evaluating the potentials and the height offsets of the Nov07, NAVD88, and CGVD28 datums using Canadian GNSS/levelling benchmark information. Additionally, it has been shown that the effect of the ellipsoidal, levelling, and geoid height errors contribute up to 4 cm to LVD offsets computed over the Canadian mainland, and can have a dm-level impact for island regions. In addition to including the accuracy information of the GNSS/levelling data and GGM coefficients, the residual geoid heights should ideally be evaluated with Stokes integration of local gravity data and using terrain data in order to account for the higher frequency components of the gravity field when utilizing the GOCE models in Canada. The inclusion of these factors is necessary when using the GNSS/levelling data over the Canadian mainland to compute the LVD offsets, and are especially important for regions with very few benchmarks, limited geographical coverage, and rugged terrain, such as independent levelling networks on islands, where the geoid model errors and the measurement errors of the GNSS/levelling heights may not average out.

Lastly, the removal of systematic effects from the height data is an essential step for the computation of LVD offsets. For the Canadian mainland GNSS/levelling network, it has been shown that both a small N-S spatial tilt and a relatively larger W-E spatial tilt exist in the Nov07 regional vertical datum, and that these components can be modelled with a 2-parameter model. Using the 2-parameter model improves the RMS of the adjusted geoid height differences by up to 4 cm though the LVD offset estimates do not change from the null model case.

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