

|| GRAVITY, GEOID AND HEIGHT SYSTEMS 2016

GOCE variance and covariance contribution to height system unification

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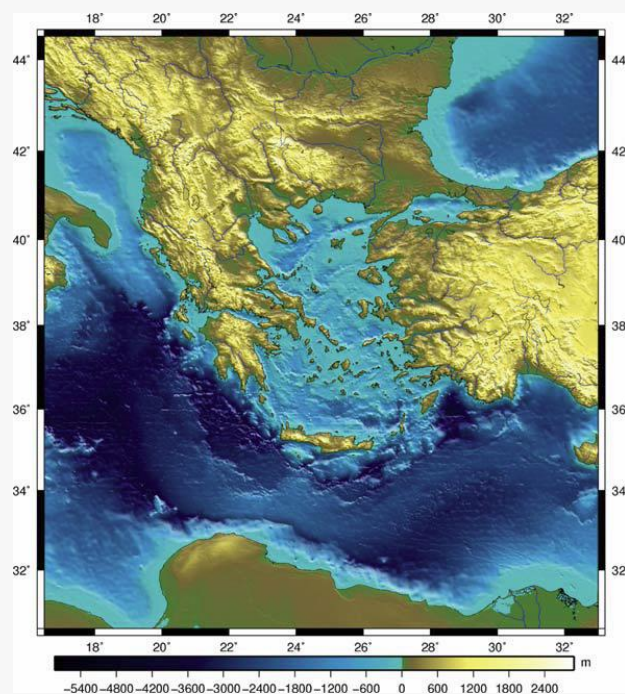
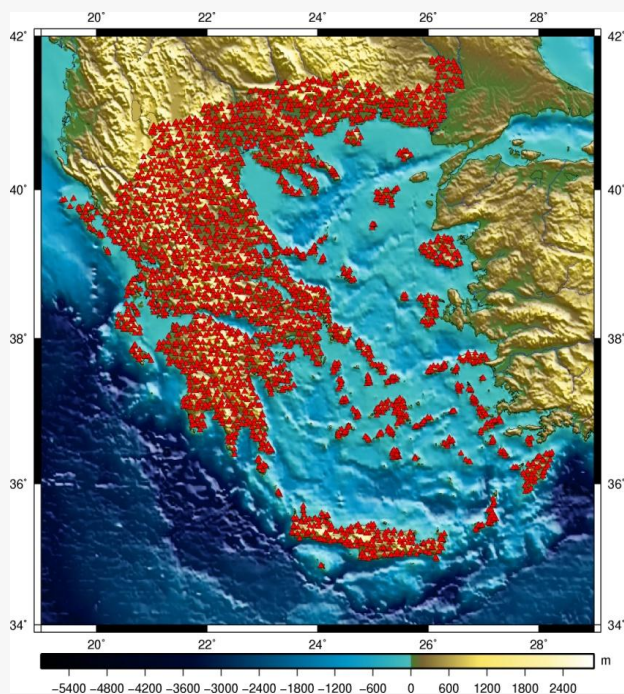
- Objectives
- Area and data description
- Methodology analysis
- Results and discussion
- Conclusions and future work

Objectives

- Investigation of the influence of GOCE errors in the determination of the Hellenic LVD
- Use of recent GOCE satellite-only and combined models
- Spectral enhancement approach for data validation
- Various approaches on the weighting of the geoid heights
- Weighting influence to the W_0 estimation

Area and Data description

- 1542 GPS/ Leveling benchmarks over the Greek mainland
- Geopotential models (EGM2008, GOCE models – DIR and TIM release 5 and GOCO satellite and combined models release 5)
- RTM effects from DTM of 3" resolution



Methodology analysis

- GPS/Leveling geoid heights were transformed to tide – free system (orthometric heights \rightarrow mean tide)
- GOCE information is taken into account to a maximum degree (175 and n_{\max})
- EGM2008 and RTM effects were subtracted \rightarrow reduced ΔN were modeled

$$\Delta N = N^{GPS/Lev} - N^i \Big|_2^{n_1} - N^{EGM2008} \Big|_{n_1+1}^{2160} - N^{RTM} - N_o$$

Methodology analysis

- Combined adjustment

$$\Delta N_i = \mathbf{a}_i^T \mathbf{x} + v_i^h - v_i^H - v_i^N$$

Deterministic parametric model

- Two ways to interpret the parametric model

1. Datum transformation model for geoid undulation

$$\Delta N_i = \Delta\alpha + \Delta X_o \cos\varphi_i \cos\lambda_i + \Delta Y_o \cos\varphi_i \sin\lambda_i + \Delta Z_o \sin\varphi_i + a\Delta f \sin^2\varphi_i$$

4-parameters (model A)
 5-parameters (model B)

2. Height-dependent corrector surfaces

$$h_i - (1 + \delta s_H)H - (1 + \delta s_N)N = \mu \longrightarrow \text{Scale factor(s) and bias}$$

$$\mathbf{a}_i^T \mathbf{x} = \mu + \delta s_H H_i + \delta s_N N_i \longrightarrow \text{3-parameters (model C)}$$

$$\mathbf{a}_i^T \mathbf{x} = \mu + \delta s_H H_i \longrightarrow \text{2-parameters (model D)}$$

$$\mathbf{a}_i^T \mathbf{x} = \mu + \delta s_N N_i \longrightarrow \text{2-parameters (model E)}$$

Methodology analysis

- **Wo estimation procedure**

$$W_o^{LVD} = W_o - \frac{\sum_1^m (h_i - H_i - N_i) g_i}{m}$$

For GOCE-based models:

$$N = N_o + N^{GOCE} \Big|_2^{175} + N^{EGM2008} \Big|_{176}^{2160}$$

or

$$N = N_o + N^{GOCE} \Big|_2^{n_{max}}$$

For EGM2008:

$$N = N_o + N^{EGM2008} \Big|_2^{2160}$$

W_o : 62 636 853.4 m²s⁻² [IAG Resolution No.1/2015]

Methodology analysis

- Weighting consideration

$$C_i = \sigma_i^2 Q_i, \quad i: h, H, N \quad \mathbf{P} = \begin{bmatrix} \mathbf{Q}_h^{-1} & 0 & 0 \\ 0 & \mathbf{Q}_H^{-1} & 0 \\ 0 & 0 & \mathbf{Q}_N^{-1} \end{bmatrix}$$

- Assumptions on the ellipsoid and orthometric height weights

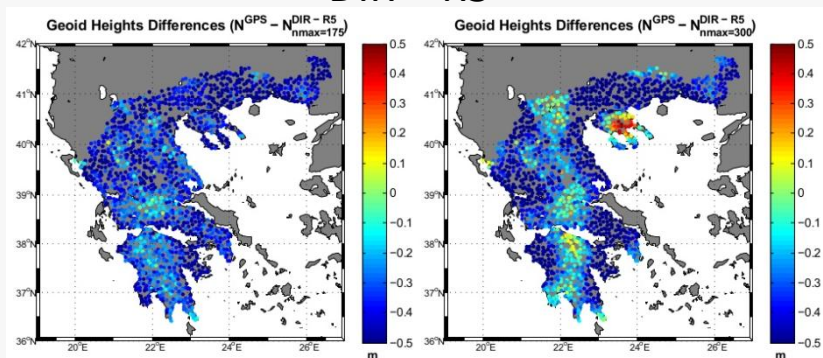
$$Q_h = \mathbf{I} \quad \text{or} \quad Q_h = 0.01 \cdot \mathbf{I} \quad Q_H = \mathbf{I} \quad \text{or} \quad Q_H = 0.04 \cdot \mathbf{I}$$

- Four different weighting methodologies for the geoid height

1. Equally weighted heights $Q_h = Q_H = Q_N = \mathbf{I}$
2. Weights based on geoid model cumulative errors
3. Weights from propagated geoid model variances
4. Weights using full variance-covariance matrix

- Differences before the parametric adjustment

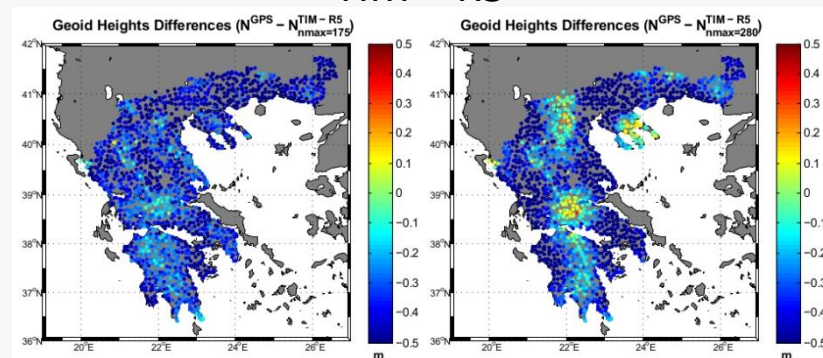
DIR – R5



$\sigma = \pm 12.9 \text{ cm}$

$\sigma = \pm 25.9 \text{ cm}$

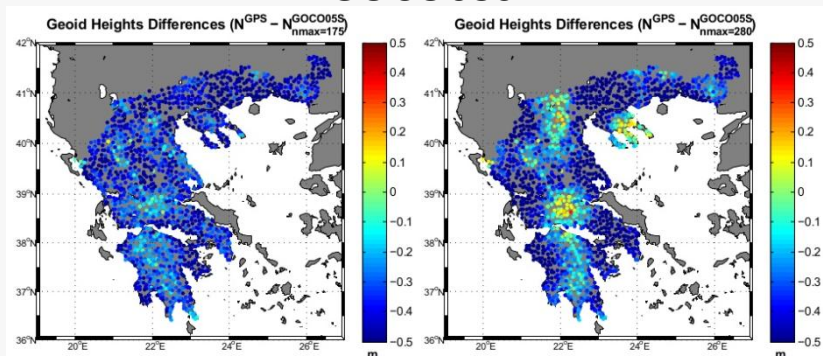
TIM – R5



$\sigma = \pm 12.7 \text{ cm}$

$\sigma = \pm 22.6 \text{ cm}$

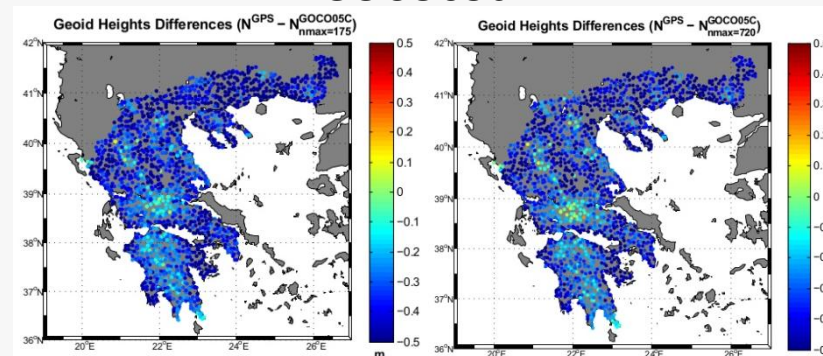
GOCO05s



$\sigma = \pm 12.7 \text{ cm}$

$\sigma = \pm 22.6 \text{ cm}$

GOCO05c



$\sigma = \pm 13.0 \text{ cm}$

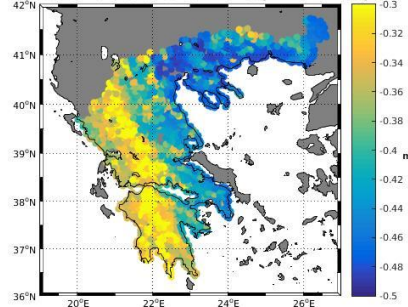
$\sigma = \pm 13.4 \text{ cm}$

- Case A: Equally weighted heights $Q_h = Q_H = Q_N = I$

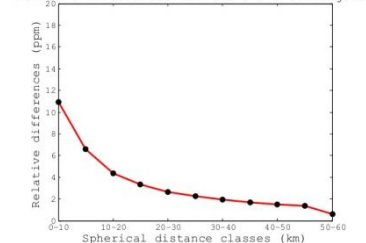
DIR – R5 (nmax = 175)

Parametric model	σ (cm)
MODEL A	± 11.9
MODEL B	± 11.9
MODEL C	± 11.3
MODEL D	± 12.1
MODEL E	± 12.2

Corrector values at benchmarks (DIR5 n=175)

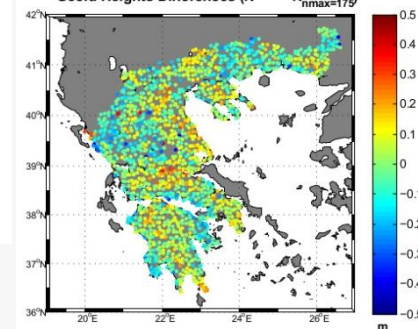


Relative differences for the Gravimetric geoid



$$a_{ij} = \frac{\Delta N_{ij}}{S_{ij}} [ppm]$$

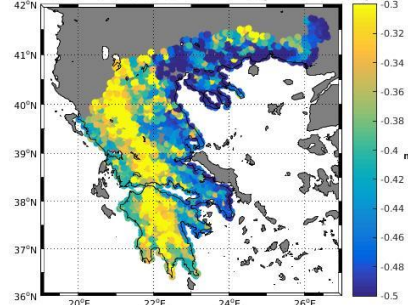
Geoid Heights Differences ($N^{GPS} - N^{DIR-R5}$ nmax=175)



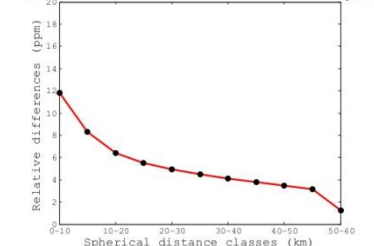
DIR – R5 (nmax = 300)

Parametric model	σ (cm)
MODEL A	± 25.5
MODEL B	± 25.5
MODEL C	± 24.4
MODEL D	± 24.8
MODEL E	± 25.6

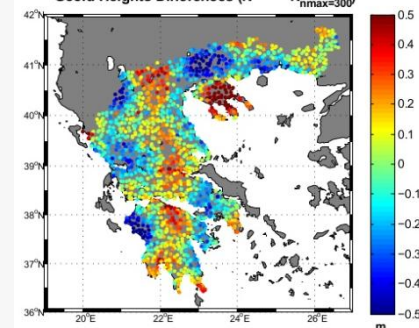
Corrector values at benchmarks (DIR5 n=300)



Relative differences for the Gravimetric geoid



Geoid Heights Differences ($N^{GPS} - N^{DIR-R5}$ nmax=300)

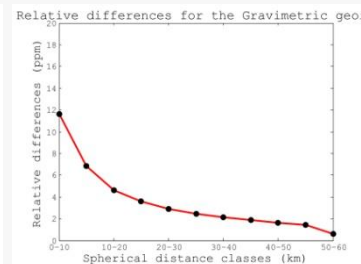
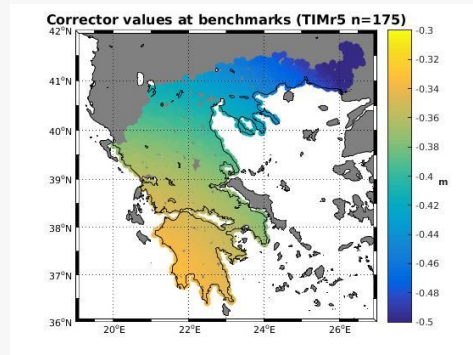


Results

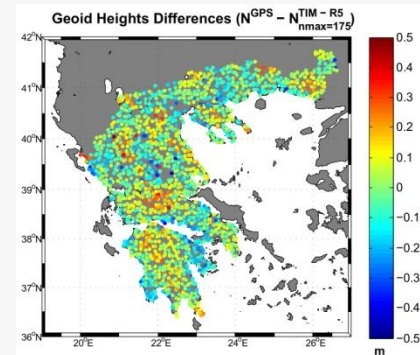
- Case A: Equally weighted heights $Q_h = Q_H = Q_N = I$

TIM – R5 (nmax = 175)

Parametric model	σ (cm)
MODEL A	± 11.8
MODEL B	± 11.8
MODEL C	± 11.1
MODEL D	± 11.9
MODEL E	± 12.1

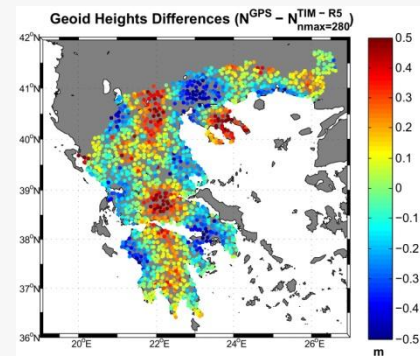
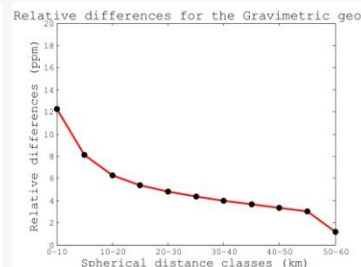
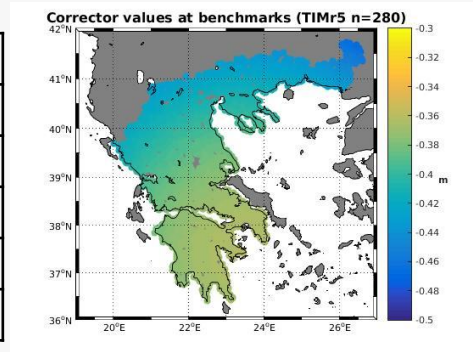


$$a_{ij} = \frac{\Delta N_{ij}}{S_{ij}} [ppm]$$



TIM – R5 (nmax = 280)

Parametric model	σ (cm)
MODEL A	± 22.4
MODEL B	± 22.4
MODEL C	± 21.4
MODEL D	± 21.2
MODEL E	± 22.4

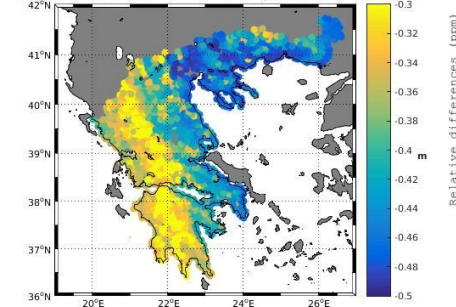


- Case A: Equally weighted heights $Q_h = Q_H = Q_N = I$

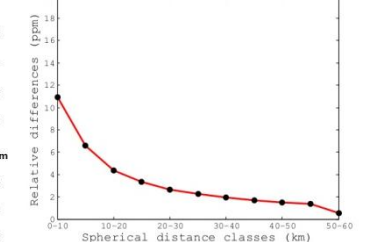
GOCO05c (nmax = 175)

Parametric model	σ (cm)
MODEL A	± 12.0
MODEL B	± 12.0
MODEL C	± 11.2
MODEL D	± 12.1
MODEL E	± 12.2

Corrector values at benchmarks (GOCO05c n=175)

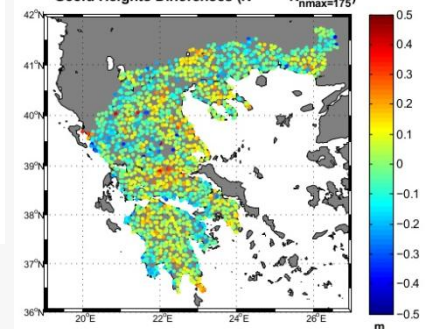


Relative differences for the Gravimetric geoid



$$a_{ij} = \frac{\Delta N_{ij}}{S_{ij}} [ppm]$$

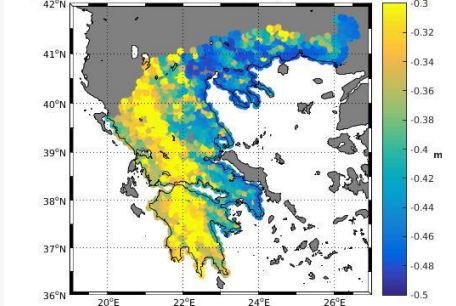
Geoid Heights Differences ($N^{GPS} - N^{GOCO05c}$, nmax=175)



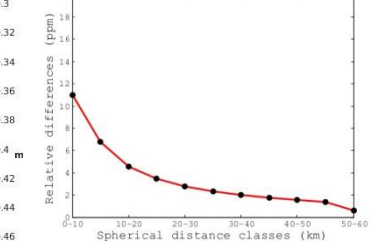
GOCO05c (nmax = 720)

Parametric model	σ (cm)
MODEL A	± 12.7
MODEL B	± 12.7
MODEL C	± 11.5
MODEL D	± 12.2
MODEL E	± 12.4

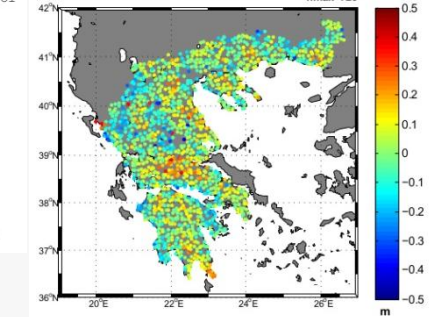
Corrector values at benchmarks (GOCO05c n=720)



Relative differences for the Gravimetric geoid



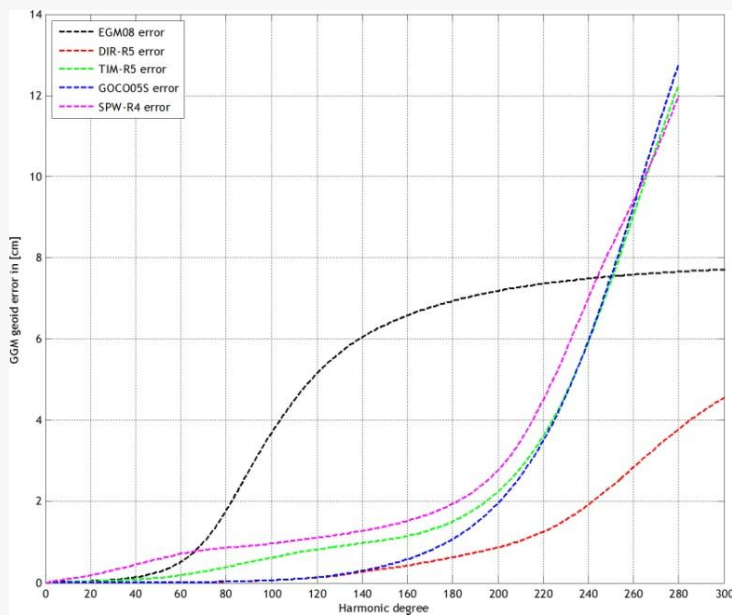
Geoid Heights Differences ($N^{GPS} - N^{GOCO05c}$, nmax=720)



Results

- Case B: Weights based on geoid model cumulative errors

$$\mathbf{Q}_N = \varepsilon_{cml}^2 \cdot \mathbf{I} = (\varepsilon_{N_{GGM} \text{ to } n_{max}}^2 + \varepsilon_{N_{08} \text{ to } 2190}^2) \cdot \mathbf{I}$$



- Same statistics in the differences with case A
- Almost identical parameters estimation

DIR – R5 (175) - MODEL C

Case A	Case B
$\hat{\mathbf{x}} = \begin{bmatrix} -0.16356 \text{ (m)} \\ 0.000106764529786 \\ -0.007659611111327 \end{bmatrix}$	$\hat{\mathbf{x}} = \begin{bmatrix} -0.16356 \text{ (m)} \\ 0.000106764529786 \\ -0.00765961111324 \end{bmatrix}$

- Similar parameters accuracy estimation

$$\hat{\sigma}_\mu = \pm 0.0185 \text{ m} \quad \hat{\sigma}_{\delta S_H} = \pm 6.51 \cdot 10^{-6} \quad \hat{\sigma}_{\delta S_N} = \pm 4.96 \cdot 10^{-4}$$

- Different a-posteriori variance estimation due to the stochastic model used in each case

$$\hat{\sigma}^2 = 0.00425 \text{ m}^2 \quad \text{Case A}$$

$$\hat{\sigma}^2 = 0.2530 \text{ m}^2 \quad \text{Case B}$$

- Same behavior to all geopotential and parametric models

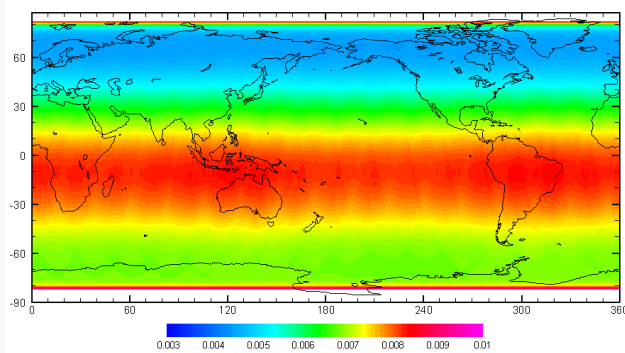
$$\varepsilon_{cml}^2 = R^2 \sum_{m=0}^n (\sigma_{\bar{c}_{nm}}^2 + \sigma_{\bar{s}_{nm}}^2)$$

Results

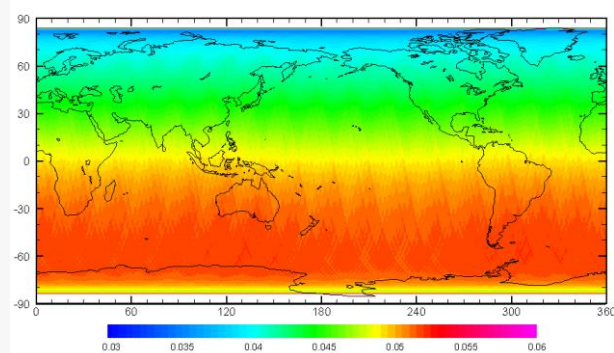
- Case C: Weights from propagated geoid model variances

$$\mathbf{Q}_N = \sigma_{prop}^2 \cdot \mathbf{I} = (\sigma_{prop}^2_{N_{GGM} \text{ to } n_{max}} + \varepsilon_{N_{08} \text{ to } 2190}^2) \cdot \mathbf{I}$$

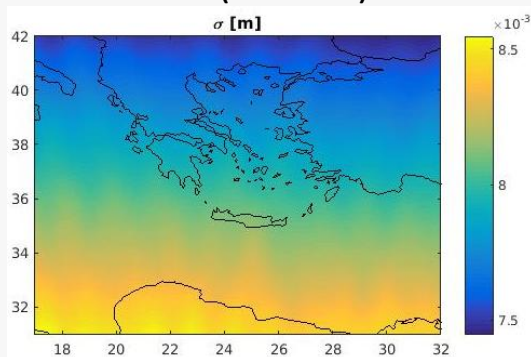
DIR – R5 (n = 175) - variances



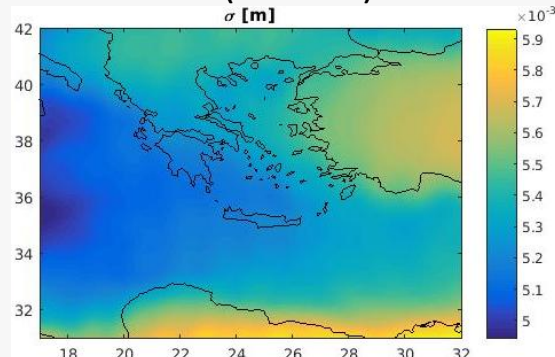
DIR – R5 (n = 300) - variances



GOCO05s (n = 175) - sd



GOCO05c (n = 175) - sd



The variance matrix was generated by bilinear interpolation at the 1542 benchmarks locations

Results

- Case C: Weights from propagated geoid model variances

$$\mathbf{Q}_N = \sigma_{prop}^2 \cdot \mathbf{I} = (\sigma_{prop}^2 N_{GGM\ to\ nmax} + \varepsilon_{N_{08\ nmax\ to\ 2190}}^2) \cdot \mathbf{I}$$

- Identical results in the statistics of the differences
- Minor differences in the parameters estimation

GOCO05s (175) – MODEL C

Case A

$$\hat{\mathbf{x}} = \begin{bmatrix} -0.18289 \text{ (m)} \\ 0.000104578589793 \\ -0.007017424236769 \end{bmatrix}$$

Case B

$$\hat{\mathbf{x}} = \begin{bmatrix} -0.18289 \text{ (m)} \\ 0.000104578589793 \\ -0.007017424236780 \end{bmatrix}$$

Case C

$$\hat{\mathbf{x}} = \begin{bmatrix} -0.18288 \text{ (m)} \\ 0.000104577983220 \\ -0.007017482757325 \end{bmatrix}$$

- Similar parameters accuracy estimation

$$\hat{\sigma}_\mu = \pm 0.0182 \text{ m} \quad \hat{\sigma}_{\delta S_H} = \pm 6.42 \cdot 10^{-6} \quad \hat{\sigma}_{\delta S_N} = \pm 4.90 \cdot 10^{-4}$$

- Different a-posteriori variance estimation

Case A

$$\hat{\sigma}^2 = 0.00413 \text{ m}^2$$

Case B

$$\hat{\sigma}^2 = 0.2453 \text{ m}^2$$

Case C

$$\hat{\sigma}^2 = 0.2467 \text{ m}^2$$

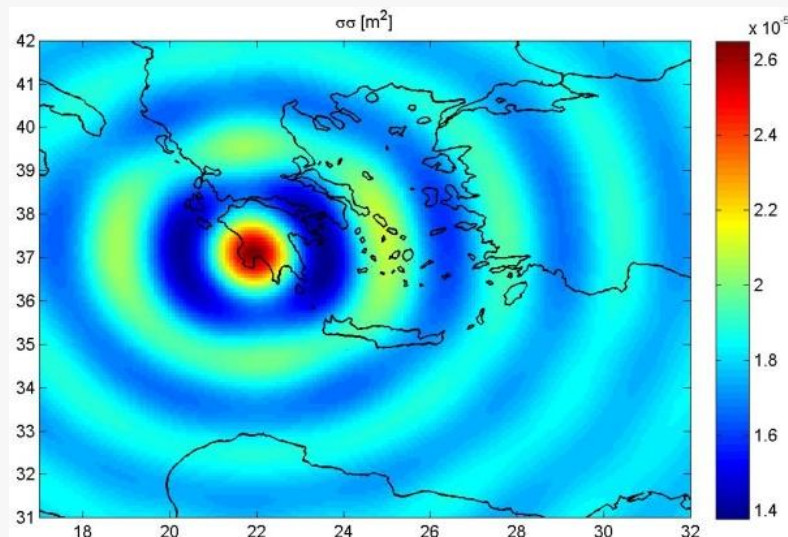
- Case D: Weights using full variance-covariance matrix

$$Q_N = C_{propN_{GGM\ to\ nmax}}^{full} + (\epsilon_{N_{08\ nmax\ to\ 2190}}^2) \cdot \mathbf{I}$$

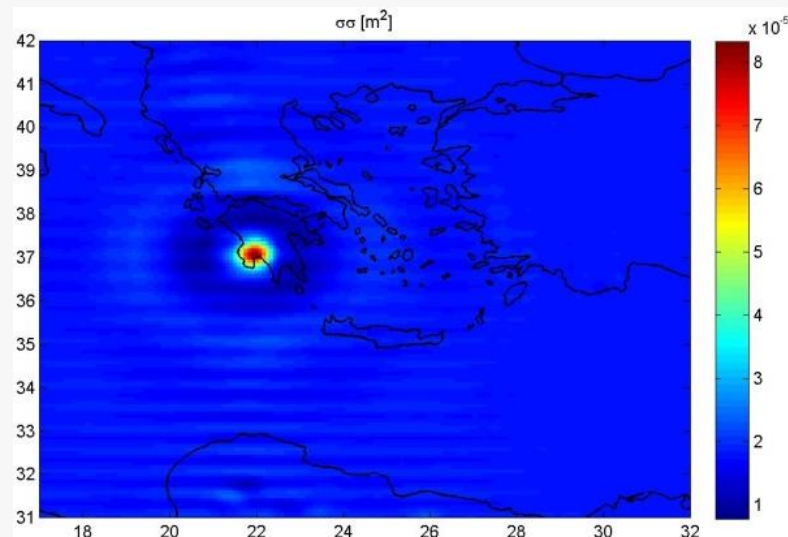
- No covariance information from nmax to 2190 → only cumulative errors used

GOCO05c – Covariance at point no 941

nmax = 175



nmax = 720



- Covariance row/column constructed by bilinear interpolation from specific covariance file

- Case D: Weights using full variance-covariance matrix

$$\mathbf{Q}_N = \mathbf{C}_{propN_{GGM\ to\ nmax}}^{full} + (\varepsilon_{N_{08\ nmax\ to\ 2190}}^2) \cdot \mathbf{I}$$

- Identical results in the statistics of the differences
- Minor differences in the parameters estimation

GOCO05c (nmax = 720) – MODEL C

Case A	Case B	Case C	Case D
$\hat{\mathbf{x}} = \begin{bmatrix} -0.18493 \text{ (m)} \\ 0.000131944657027 \\ -0.007113661382659 \end{bmatrix}$	$\hat{\mathbf{x}} = \begin{bmatrix} -0.18493 \text{ (m)} \\ 0.000131944657027 \\ -0.007113661382667 \end{bmatrix}$	$\hat{\mathbf{x}} = \begin{bmatrix} -0.18518 \text{ (m)} \\ 0.000132107484412 \\ -0.007107377090662 \end{bmatrix}$	$\hat{\mathbf{x}} = \begin{bmatrix} -0.18605 \text{ (m)} \\ 0.000133008275695 \\ -0.0071085682762273 \end{bmatrix}$

- Similar parameters accuracy estimation

$$\hat{\sigma}_{\mu} = \pm 0.0006 \text{ m} \quad \hat{\sigma}_{\delta S_H} = \pm 0.21 \cdot 10^{-6} \quad \hat{\sigma}_{\delta S_N} = \pm 0.16 \cdot 10^{-4}$$

- Different a-posteriori variance estimation

Case A	Case B	Case C	Case D
$\hat{\sigma}^2 = 0.0044 \text{ m}^2$	$\hat{\sigma}^2 = 0.1994 \text{ m}^2$	$\hat{\sigma}^2 = 0.2629 \text{ m}^2$	$\hat{\sigma}^2 = 0.2574 \text{ m}^2$

Results

• $W_o^{LVD} - W_o^{IAG}$ [Mainland of Greece]

Weighting Scheme	EGM08	GOCO05C / EGM08	GOCO05s / EGM08	DIR R5 / EGM08	TIM R5 / EGM08
None (average)	6.264 ±0.035	6.409 ±0.035	6.443 ±0.034	6.414 ±0.034	6.459 ±0.034
$1 / (\sigma_h^2 + \sigma_H^2 + \sigma_N^2)$ [commission error]	<i>No change in results</i>				
$1 / (\sigma_h^2 + \sigma_H^2 + \sigma_N^2)$ [GOCE variances + EGM08 com error]	<i>No change in results</i>				
$1 / (\sigma_h^2 + \sigma_H^2 + \sigma_N^2)$ [GOCE var/covar + EGM08 com error]		-	6.401 ±0.039		

Units: m^2s^{-2}

- Variance Component Estimation

$$C_i = \sigma_i^2 Q_i, \quad i: h, H, N$$

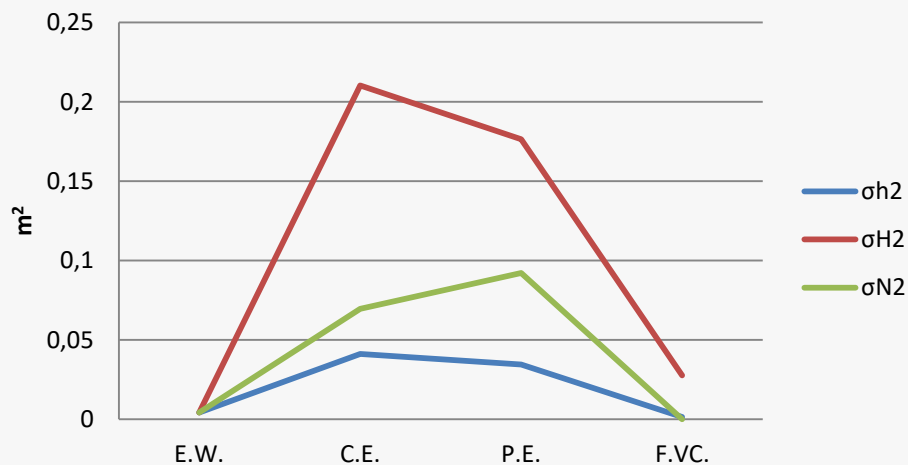
- Unknown variance components using iterative MINQUE (Rao, 1971, 1977)

GOCO05c (nmax = 720)

Initial values	Equally weighted heights	Geoid variances from model cumulative errors	Geoid variances from propagated errors	Full variance-covariance geoid information
$\sigma_h^2 = \sigma_H^2 = \sigma_N^2 = 1$	$\hat{\sigma}_h^2 = 0.00438 \text{ m}^2$ $\hat{\sigma}_H^2 = 0.00438 \text{ m}^2$ $\hat{\sigma}_N^2 = 0.00438 \text{ m}^2$	$\hat{\sigma}_h^2 = 0.04119 \text{ m}^2$ $\hat{\sigma}_H^2 = 0.21030 \text{ m}^2$ $\hat{\sigma}_N^2 = 0.06968 \text{ m}^2$	$\hat{\sigma}_h^2 = 0.03450 \text{ m}^2$ $\hat{\sigma}_H^2 = 0.17642 \text{ m}^2$ $\hat{\sigma}_N^2 = 0.09215 \text{ m}^2$	$\hat{\sigma}_h^2 = 0.00146 \text{ m}^2$ $\hat{\sigma}_H^2 = 0.02779 \text{ m}^2$ $\hat{\sigma}_N^2 = 6.76 \cdot 10^{-8} \text{ m}^2$
$\sigma_h^2 = 0.01 \text{ m}^2$ $\sigma_H^2 = 0.04 \text{ m}^2$ $\sigma_N^2 = 1$		$\hat{\sigma}_h^2 = 0.06572 \text{ m}^2$ $\hat{\sigma}_H^2 = 0.26653 \text{ m}^2$ $\hat{\sigma}_N^2 = 0.10652 \text{ m}^2$	$\hat{\sigma}_h^2 = 0.05627 \text{ m}^2$ $\hat{\sigma}_H^2 = 0.22830 \text{ m}^2$ $\hat{\sigma}_N^2 = 0.09155 \text{ m}^2$	$\hat{\sigma}_h^2 = 0.04253 \text{ m}^2$ $\hat{\sigma}_H^2 = 0.17217 \text{ m}^2$ $\hat{\sigma}_N^2 = 0.01257 \text{ m}^2$

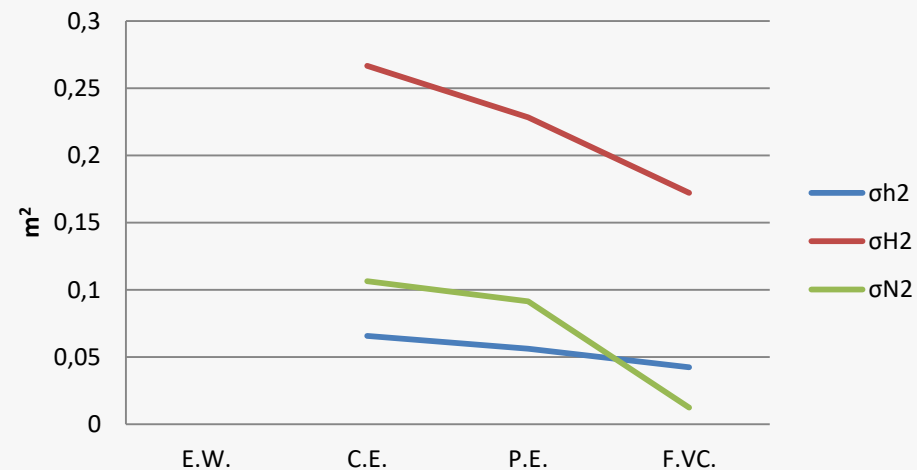
• Variance Component Estimation

VCE - Initial values $\sigma^2 = 1$



E.W.: Equally weighted heights
 C.E.: Weights from cumulative geoid errors

VCE - Initial values $\sigma^2 = [0.01 \ 0.04 \ 1]^T$



P.E.: Weights from propagated geoid errors
 C.E.: Full variance-covariance geoid matrix

Conclusions and future work

- The weighting effect
 - a) on the **residual geoid modeling** using parametric models,
 - b) on the **variance component estimation** and
 - c) on the **local W_0 estimation** is investigated.
- Minor differences in the parameter estimation were revealed.
- The different a-posteriori variance of each solution is depending on the specific scenario used for the stochastic model (weighting cases).

Conclusions and future work

- The adequateness of the parametric model utilized has to be confirmed applying statistical tests for the parameters.
- The use of the full variance/covariance matrix for the geoid heights led to a decrease in variance component estimation values.
- In all weighting schemes VCE led to an increased error for the orthometric heights, signaling that the Greek LVD is outdated and needs modernization (geoid-based datum?)

Conclusions and future work

- W_0 estimation with GOCO05s, GOCO05c, TIM-R5, DIR-R5 patched with EGM08 leads to similar results for the Greek Mainland (differences up to $0.05 \text{ m}^2\text{s}^{-2}$)
- W_0 estimation with GOCE-based models patched with EGM08 versus EGM08 leads to significantly different results ($0.15 - 0.19 \text{ m}^2\text{s}^{-2}$)
- Use of error variances or error variances/covariances matrix of the GOCO models in the adjustment weighting scheme made no change to the W_0 estimates.

Acknowledgment

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