Adjustment of collocated GPS, geoid and orthometric height observations in Greece. Geoid or orthometric height improvement?

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Abstract. The combined adjustment of GPS/Levelling observations on benchmarks with gravimetric geoid heights has been the focus of extensive research both from the theoretical and practical point of view. Up until today, with few exceptions, the main blame for the inconsistencies/disagreement between these three types of heights has been put to the geoid heights due mainly to their poorer accuracy. With the advent of the new CHAMP- and GRACE-based global geopotential models and the realization of EGM2008 the achievable cumulative geoid accuracy has improved significantly so that its differences to GPS/Levelling heights reach the few cm level. In Greece, GPS observations on BMs are very scarce and cover only small parts, in terms of spatial scale, of the country. Recently, an effort has been carried out to perform new GPS measurements on levelling BMs, so that reliable GPS/Levelling and gravimetric geoid height adjustment studies can be carried out. This resulted in part of North-Western Greece to be covered with reliable observations within an area extending 3 degrees in longitude and 1 degree in latitude. Therefore, some new potential for the common adjustment of the available geometric, orthometric and geoid heights, using various parametric surfaces to model and interpret their differences, are offered. These are used to come to some conclusions on the accuracy of the various geoid models used (both global geopotential and local gravimetric models), while an extensive outlook is paid to the questionable behaviour of the orthometric heights. The latter is especially important for the Greek territory since the available benchmarks are delaminated in so-called "map-leaflets" and a common adjustment of the entire vertical network has not been carried out so far. It is concluded that even between neighbouring "map-leaflets" large biases in the adjusted GPS/Levelling and gravimetric geoid heights exist, which indicates distortions in the Greek vertical datum as this is realized by the levelling benchmarks. Given that the latter are commonly used for everyday surveying purposes, conclusions and proposals on the determination of adjusted orthometric heights are finally drawn.

Keywords. EGM2008, geoid, GPS, height adjustment, vertical datum, parametric models.

1 Introduction

During the last two decades and since the advent of GNSS positioning, the combined adjustment of GPS ellipsoidal heights (h) with orthometric heights (H) from conventional levelling and gravimetric geoid heights (N) has gained increasing importance (Featherstone 1998). This refers both to the scientific treatment of the combination problem as well as to every day surveying campaigns. The purely scientific treat of the combination of these three height types dealt mainly with efforts to model and interpret the height residuals at stations where collocated GPS/Levelling and geoid observations were available. The differences between them were, and still are, explained as datum biases, long-wavelength geoid errors and random errors remaining to all height types. In most cases, the blame for the large discrepancies was put to gravimetric geoid heights due to the inadequacy, in both resolution and accuracy, of the historical gravimetric databases and the unavailability of satellite observations to boost the accuracy of global geopotential models (GGMs) to higher degrees of expansion. On the other hand, GPS and levelling observations were considered to contribute little to the total error budget due to the accuracy of the former in differential static measurements at levelling benchmarks (BMs) and the unanimous knowledge that spirit levelling is indeed the most accurate means for orthometric height determination. Within this frame, collocated observations of h, H and N are used to: a) assess the external accuracy of gravimetric geoid models (Featherstone et al. 2001), b) construct so-called corrector surfaces in an area of study, so that the transformation between either of the three can be made (Sideris et al. 1992), and c) substitute conventional spirit levelling by GPS/Levelling during which there is no need to measure orthometric heights since they are determined by GPS measurements and gravimetric geoid heights (Fotopoulos et al. 2001; Vergos and Sideris 2002). A distinction has to be made at this point concerning the terms *scientific* and *everyday surveying* purposes mentioned previously. As far as the former is concerned, we are mostly interested in the absolute differences between h, H and N using statistical measures as the range, mean and standard deviation (std) to assess the performance of (mainly) the available gravimetric geoid model and/or GGM. Relative differences are important as well, but as an additional measure of the achievable accuracy. Due to the need for high-accuracy in an absolute sense, almost all available GGMs and gravimetric geoid models, until recently, did not manage to provide rigorous results for point c) above. On the other hand, for everyday surveying purposes, where a pair of GPS receivers is used and the base is set at a reference benchmark, the need for high absolute accuracy is not mandatory. This is so because even with an EGM96-class of GGM, the long-wavelength and other errors in the geoid heights are removed by computing essentially relative height differences between the measuring point that the rover and the benchmark of the base is set to.

With the recent gravity-field dedicated missions of CHAMP, GRACE and GOCE and the realization of EGM2008 (Pavlis et al. 2008), the available GGMs have much more power up to very-high degrees and increasing accuracy. EGM2008 was released to public by the U.S. Geospatial-Intelligence Agency (NGA) EGM Development Team and presents a spherical harmonics expansion of the geopotential to degree and order 2159. The availability of such GGMs poses new potentials in order to validate available orthometric heights and subsequently correct blunders in the levelling databases. This is of special importance in countries like Greece where: a) the vertical reference network, realized through the network of levelling BMs, has not been commonly adjusted in a unified frame, b) in various parts of the country the zero-point w.r.t. which the heights of the BMs have been determined, varies and is set to coincide with a local tide-gauge station, c) the levelling BMs are delaminated in so-called "map-leaflets" which often have horizontal and vertical distortions. The latter creates significant problems to everyday GPS surveying applications when levelling BMs from neighbouring "map-leaflets" are used in a single traverse. The main goal of the present study stems from the aforementioned problems for the Greek territory and has two main goals. The first one is to investigate whether blunders in the orthometric heights can be identified and corrected when collocated GPS and geoid observations are available. The second one is to evaluate the performance of GGMs and regional gravimetric geoid models in terms of the differences between h, H and N during their combined adjustment. For that purpose recent observations collected over Northern Greece in a network of 43 benchmarks (see Figure 1) are used.

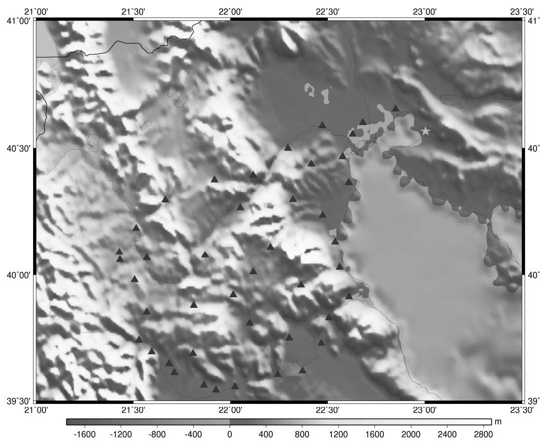

Figure 1: The distribution of the available GPS/Levelling BMs in Northern Greece (triangles).

2 Data and observation equations

Given the availability of collocated GPS, levelling and gravimetric geoid heights one can write the vector of observations ℓ_i and the observation equations for their combined adjustment as:

$$\ell_i = h_i - H_i - N_i^{gr} = N_i^{GPS/Lev} - N_i^{gr} \tag{1}$$

and

$$\ell_i = \mathbf{a}_i^T \mathbf{x}_i + v_i \ . \tag{2}$$

where the elements a_i^T of the design matrix $\boldsymbol{A}$ and the unknowns x_i depend on the parametric model chosen to describe the differences between the triplet of heights. In Eqs. (1) and (2), h_i, H_i and N_i^{gr} denote the available GPS, levelling and gravimetric geoid heights at station i, and $N_i^{GPS/Lev} = h_i - H_i$ are the so-called GPS/Levelling geoid heights. For the parametric model to be used, various choices have been tested, namely the well-known four- and five-parameter similarity transformation models and 1[st], 2[nd] and 3[rd] order polynomial ones, as presented in Eqs. (3)-(5) respectively (Fotopoulos 2003)

$$a_i^T x = x_0 + x_1 \cos\varphi_i \cos\lambda_i + x_2 \cos\varphi_i \sin\lambda_i + \\ + x_3 \sin\varphi_i \ , \tag{3}$$

$$a_i^T x = x_0 + x_1 \cos\varphi_i \cos\lambda_i + x_2 \cos\varphi_i \sin\lambda_i + \\ + x_3 \sin\varphi_i + x_4 \sin^2\varphi_i \ , \tag{4}$$

$$a_i^T x = \sum_{m=0}^{M} \sum_{n=0}^{N} x_q (\varphi_i - \varphi_0)^n (\lambda_i - \lambda_0)^m \cos^m \varphi_i \ . \qquad (5)$$

In matrix notation the system of observation equations and the solution are written

$$\mathbf{b} = \mathbf{Ax} + \mathbf{v} \qquad (6)$$

and

$$\hat{\mathbf{x}} = \left(\mathbf{A}^T \mathbf{PA}\right)^{-1} \mathbf{A}^T \mathbf{Pb} \ . \qquad (7)$$

In Eq. (7), matrix ***P*** is the weight matrix, i.e., the inverse of the variance-covariance matrix ***C*** of the observations. Throughout this study we have assumed that a) the observations and the errors are uncorrelated for all height types and b) no correlation exists for the same height type among different observation stations *i*. Therefore the minimization principle and the corresponding weight matrix take the form (Kotsakis and Sideris 1999):

$$\mathbf{P} = \left(\mathbf{C}_{h^{GPS}} + \mathbf{C}_{H^{LEV}} + \mathbf{C}_{N^{grav}}\right)^{-1} \qquad (8)$$

and

$$\mathbf{v}_{h^{GPS}}^T \mathbf{C}_{h^{GPS}}^{-1} \mathbf{v}_{h^{GPS}} + \mathbf{v}_H^T \mathbf{C}_H^{-1} \mathbf{v}_H + \mathbf{v}_{N^{grav}}^T \mathbf{C}_{N^{grav}}^{-1} \mathbf{v}_{N^{grav}} = \min \qquad (9)$$

where ***v*** and ***C*** denote residuals and variance-covariance matrices of the GPS, levelling and gravimetric geoid height observations. Based on the parameter estimation in Eq. (7), adjusted observations $\hat{\mathbf{h}}, \hat{\mathbf{H}}, \text{and}\ \hat{\mathbf{N}}_{grav}$ can be estimated as well along with adjusted residuals $\hat{\mathbf{v}}$ and adjusted variance-covariance matrices $\hat{\mathbf{C}}_h, \hat{\mathbf{C}}_H, \text{and}\ \hat{\mathbf{C}}_N$ (see Fotopoulos 2003).

Within the frame of the objectives set, first an evaluation of the available parametric models is performed using EGM2008 geoid heights in order to determine the one that provides the best fit. The one selected, is then employed to detect blunders in the orthometric heights and estimate new corrected values. A new adjustment using these corrected orthometric heights is performed in order to assess the improvement achieved. Then, an investigation of the influence of the observation input errors on the results of the adjustment is carried-out. Therefore, the fit achieved, when using the local gravimetric geoid model and the other GGMs, is compared to the results provided by EGM2008. The GGMs employed in this study in order to investigate their fit to the GPS/Levelling geoid heights are EGM2008 (Pavlis et al. 2008), EGM96 (Lemoine et al. 1998), GGM03c, GGM03s (Tapley et al. 2007) and EIGEN5c (Reigber et al. 2005) representing the latest satellite-only and combined models. The final part is devoted to some examples of the biases that exist between neighbouring "map-leaflets" in the adjusted GPS/Levelling and gravimetric geoid heights.

3 Combined adjustment results

The first set of tests deals with the improvement that each parametric model offers in the adjusted height residuals. All five models have been tested employing the 43 GPS/Levelling observations, geoid heights from EGM2008 and a uniform accuracy of ±1 cm for all height types. It is worth mentioning that higher-order polynomial models have also been tested but their parameters have been proven insignificant. From Table 1, where the results are summarized, it becomes evident that the best fit is achieved when the 3rd order polynomial model is employed to model the residuals. After the fit, a reduction by 8 cm of the std is achieved while the range reduces also by ~66 cm. The performance of the 3rd order polynomial model is 1-5 cm better (1σ) than the others, which gives good evidence that it is the one to be used for all subsequent fit investigations. Examining the residuals before the fit, the large mean and std of the height differences is noticing. Even though the mean can be attributed to some datum bias, which is treated by the parametric model, the std of the differences is outside the range of the performance of EGM2008, at least for European areas. The latter is expected to reach ~16-17 cm according to the EGM2008 validation performed during its development (Pavlis et al. 2008).

Table 1: Statistics of the differences $N^{GPS/Lev}$-N^{EGM08} before and after the fit. Unit: [m]

	max	**min**	**mean**	**rms**	**std**
before	1.314	-0.268	0.750	±0.786	±0.234
4-param	0.456	-0.649	0.000	±0.178	±0.178
5-param	0.360	-0.634	0.000	±0.168	±0.168
1st pol.	0.400	0.813	0.000	±0.200	±0.200
2nd pol.	0.340	-0.619	0.000	±0.163	±0.163
3rd pol.	0.320	-0.598	0.000	±0.156	±0.156

Plotting the height differences for all stations (see Fig. 2), the results achieved for two of these (pointed with a circle in Fig. 2) indicate that they probably contain blunders. This was concluded based on a 2rms criterion applied to the residuals before the fit (see 1st line in Table 1). Given that the EGM2008 accuracy can be regarded uniform for small areas like the one under investigation and that no blunders are included in the GPS geometric heights, the blame can be put to the orthometric heights for the benchmarks under question. In order to computed adjusted orthometric heights for the two BMs, a new fit was carried out, using the remaining 41 stations and a 3rd order polynomial as a parametric model. Then, employing Eq. (11), adjusted orthometric heights have been determined by applying corrections of 0.506 m and 0.115 m. Following the determination of the adjusted orthometric heights a new common adjustment of all 43 stations, similar to the previous one, has been carried out with the results reported in Table 2. Comparing the residuals before the fit (first line in Tables 1 and 2), when the new adjusted orthometric heights are employed, an improvement in the std by ~6 cm is achieved. This signals that the estimated adjusted orthometric heights for the two BMs successfully manage to provide smaller residuals. Moreover, the initial "formal" values for the heights of the levelling benchmarks clearly contain errors which would be propagated to any surveying observations if used. This is important too when a validation of a gravimetric geoid model is performed with such faulty orthometric height observations, since the conclusions drawn would be misleading. In any case, from the results presented in Table 2, the superior performance of the 3rd order polynomial model is once again evident, since the std drops by ~5 cm compared to the differences before the fit and the range by ~79 cm. Notice that the incorporation of the adjusted orthometric heights for the two BMs improves the fit as well, since the std and the range after the fit with the 3rd order polynomial model improve by 3.3 cm and 35 cm respectively (last row in Tables 1 and 2).

Table 2: Statistics of the differences $N^{GPS/Lev}$-N^{EGM08} before and after the fit using the adjusted orthometric heights for the two BMs. Unit: [m]

	max	min	mean	rms	std
before	1.118	0.238	0.741	±0.786	±0.176
4-param	0.337	-0.338	0.000	±0.137	±0.137
5-param	0.319	-0.372	0.000	±0.135	±0.132
1st pol.	0.323	-0.389	0.000	±0.160	±0.160
2nd pol.	0.291	-0.387	0.000	±0.135	±0.135
3rd pol.	0.244	-0.324	0.000	±0.123	±0.123

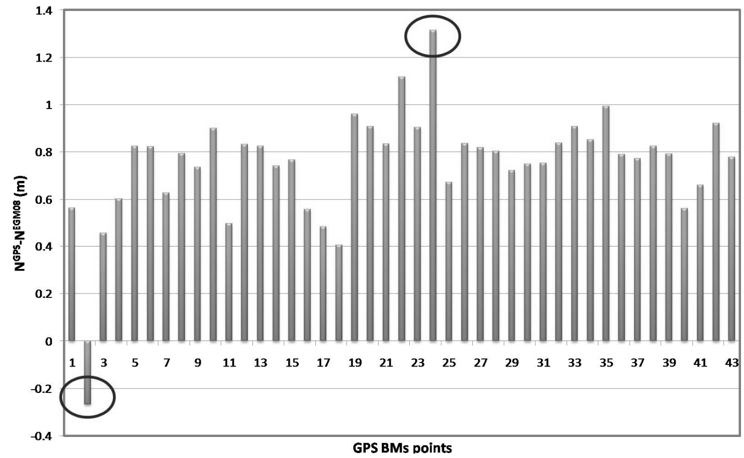

Figure 2: Differences between GPS/Levelling and EGM2008 geoid heights at available BMs.

The next set of tests performed refers to the investigation of the influence that the data input error would have on the adjusted residuals. To this extent three separate cases have been identified: a) The first one assumes that all height types have a uniform accuracy of ±1 cm, so that the covariance matrices are all equal to the identity matrix ***I***, b) A-priori standard deviations (σ_h=±2 *cm*, σ_H= ±3 *cm* and σ_N= ±4 *cm*) are assigned to the observations assuming that the accuracy of the geometric heights is the highest, with the orthometric and geoid heights following, and c) The input error for the geometric heights was that from the GPS data processing, the error of the orthometric heights was the formal one provided by the Hellenic Military Geographic Service and the geoid height error was set again to a standard deviation σ_N= ±4 *cm*. All these cases will be identified herein as *caseA*, *caseB* and *caseC* respectively. It should be noted that the mean error for *caseC* was ±0.3 cm and ±0.5 cm for the ellipsoidal and orthometric heights, respectively. In all cases the adjustment took place by employing the 3rd order polynomial model, which provided the best results in the previous test, and geoid heights from EGM2008 to represent the gravimetric geoid model. Table 3 presents the results achieved after the fit for the three scenarios examined. It is clear that no improvement is achieved when employing the most rigorous *caseC* for the data covariance matrices, even compared to *caseA* where the input errors are set equal to ±1 cm for all height types. The reduction of the std of the differences by 1 mm for *caseC* is clearly insignificant and signals that, as far as the fit is concerned, the input errors for the observations seem to play little role. This is of course not the case when testing and scaling the supplied covariance matrices, calibrating geoid error models and assessing/evaluating the accuracy of the orthometric heights. In such cases the input errors and variance component estimation can prove a useful and significant tool (Fotopoulos 2003).

Table 3: Statistics of the differences $N^{GPS/Lev}$-N^{EGM08} before and after the fit using different input error models. Unit: [m]

	max	min	mean	rms	std
before	1.118	0.238	0.741	±0.786	±0.176
caseA	0.244	-0.324	0.000	±0.123	±0.123
caseB	0.244	-0.324	0.000	±0.123	±0.123
caseC	0.243	-0.324	0.000	±0.122	±0.122

The final set of tests performed, incorporated the other available GGMs as well as a local gravimetric geoid model developed for the Greek territory. The objectives were twofold. First to investigate and assess the improvement that EGM2008 brings compared to older GGMs and secondly to determine its performance w.r.t. a local geoid model. A brief overview of the latter, with emphasis on the treatment of the topographic effects is given in Tziavos et al. (2009). Once again a 3rd order polynomial model has been employing to describe the differences between ellipsoidal, orthometric and geoid heights while *caseC*, the most rigorous of the three, has been used to describe their errors. Table 4 presents the results acquired for all geoid models, both before and after the fit, with the ones for EGM2008 reported in Tables 2 and 3. From the results presented in Table 4 it is clear that EGM2008 outperforms all other GGMs, since the std of ±12.2 cm it provides after the fit is ~13 cm better than that of the others. Of course, this is expected since GGM03s is a satellite only model, while the others are complete to degree and order 360, rather than 1834 where EGM2008 was truncated. This is a clear indication of the significant improvement that this recently released GGM brings to all geosciences and especially geodetic and oceanographic research. One further note for the superior performance of EGM2008 is the std of the differences before the fit (±17.6 cm) which is better that the std of the fitted residuals for the other models. Comparing the performance of the local gravimetric geoid model, it can be concluded that it gives better results than EGM2008 by ±4 cm and ±2 cm (1σ) before and after the fit, respectively. Moreover, the range of the differences for the local gravimetric geoid model is smaller by ~42 cm and ~12 cm before and after the fit. This is a good indication that even in the presence of high-resolution and high-accuracy GGMs, like EGM2008, local and regional gravimetric geoid models have still to offer and need not to be abandoned.

Table 4: Statistics of the differences between GPS/levelling and geoid heights from the local model and the GGMs before and after the fit. Unit: [m]

	max	min	mean	rms	std
differences with $N^{grav\ local}$					
before	-0.220	-0.714	-0.452	0.471	±0.133
after	0.198	-0.237	0.000	0.104	±0.104
differences with N^{GGM03c}					
before	1.256	-0.423	0.159	0.408	±0.376
after	0.772	-0.336	0.000	0.255	±0.255
differences with $N^{EIGEN5c}$					
before	1.209	-0.603	0.040	0.378	±0.376
after	0.771	-0.317	0.000	0.252	±0.252
differences with N^{GGM03s}					
before	2.413	-1.953	-0.212	1.709	±1.438
after	0.830	-0.433	0.000	0.268	±0.268
differences with N^{EGM96}					
before	0.860	-0.784	-0.124	0.383	±0.362
after	0.758	-0.293	0.000	0.250	±0.250

A final note refers to some examples of the biases that exist between neighbouring "map-leaflets" in the adjusted GPS/Levelling and gravimetric geoid heights for the Greek levelling network. Table 5 presents the mean and std of the differences between GPS/Levelling and N^{EGM08}/$N^{gravlocal}$ geoid heights for neighbouring "map-leaflets". Note that in principle trigonometric BMs between neighbouring "map-leaflets" can be used in everyday surveying applications as known stations for traverses, so that any datum shifts between them will introduce unrealistic miss-closure errors. From Table 5, where the different "map-leaflets" are distinguished by their id, it can be concluded that significant biases ranging from 5-25 cm exist between levelling BMs residing in neighbouring "map-leaflets", which is a clear indication that, un-modelled, datum shifts exist in the Greek datum. The differences in the std range between 0.8-3 cm which can be regarded as normal as far as random errors in the vertical datum are concerned, especially for long-levelling traverses (the shortest distance between the levelling BMs for neighbouring "map-leaflets" is ~30 km in the present study). In any case, a safe conclusion can be drawn at this point, i.e., that since a common adjustment of the entire Greek vertical network has not been carried out, traverses employing BMs from more than one "map-leaflet" should be dealt with care.

Table 5: Statistics of the differences between GPS/levelling and geoid heights from the local model and EGM2008 for neighbouring map-leaflets. Unit: [m]

	mean	**std**	**mean**	**std**
map id	**132**		**12**	
N^{EGM08}	0.748	±0.118	0.816	±0.084
$N^{grav\ local}$	0.247	±0.109	0.445	±0.058
map id	**85**		**166**	
N^{EGM08}	0.848	±0.060	0.604	±0.084
$N^{grav\ local}$	0.478	±0.050	0.318	±0.042
map id	**26**		**303**	
N^{EGM08}	0.540	±0.141	0.298	±0.166
$N^{grav\ local}$	0.092	±0.112	-0.103	±0.129

4 Conclusions

A detailed scheme for the combined adjustment of ellipsoidal, orthometric and geoid heights over a network of 43 benchmarks in Greece has been presented. Various parametric models were tested, in order to model the residual differences, along with different choices for the data input errors.

From the results acquired, it was concluded that orthometric height validation and blunder detection is feasible when high-accuracy GGMs and local geoid models are available. When blunders are detected and adjusted orthometric heights are determined then improved residuals by ~6 cm are achieved. These can then be used to improve local gravimetric geoid fit to GPS/Levelling heights. In all cases the selection of a 3rd polynomial as a parametric model provided the best results for the fitted residuals, since it reduced the std and the range, compared to the other models, by ~5 cm and ~79 cm respectively. From the analysis of the influence of the errors of the observations, it was concluded that practically no improvement in the fitted residuals was achieved when either the identity or proper covariance matrices were employed. This conclusion holds for the specific set of tests and not when, e.g., the calibration of the covariance matrices is needed. EGM2008 provided the best fit when compared to the other recent GGMs signalling the significant improvement that this model brought to modern-day geodetic research. It is worth mentioning that even the std of the differences, before the fit, that EGM2008 provided was smaller than that of the other GGMs after the fit of the parametric model. Regional and local gravimetric geoid model development has still to offer, since it provided better results by ~3 cm (1σ) compared to EGM2008, which provides evidence that even ultra-high degree GGMs, at least until today, cannot depict the local peculiarities of the Earth's gravity field. Finally, some problems arising from the fact that the Greek vertical network has not been commonly adjusted for the entire country have been demonstrated. This lack of a common adjustment introduces significant biases in the orthometric heights of the order of 5-25 cm when levelling BMs from neighbouring "map-leaflets" are used in a traverse. Therefore, such operations should be exercised with caution and control.

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